Met. O. 902

METEOROLOGICAL OFFICE

Scientific Paper No. 38

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The Psychrometer Coefficient of the Wet-bulb Thermometers used in the Meteorological Office Large Thermometer Screen

By C. K. Folland, B.Sc.

SUMMARY

Experimental evidence has been accumulating which suggests that the psychrometer coefficients of the mercury-in-glass and 4-inch platinum resistance thermometers used as wet bulbs in the Meteorological Office large thermometer screen are not the same. These coefficients have now been determined for a range of ventilation rates using an automatic dew-point hygrometer as reference. The experimental values are discussed in the light of recent work in Japan and Belgium. Recommendations are made on how the observing procedure might be modified and proposals for further experimental work are made.

The suggestion is made in Part I that the psychrometer coefficient of the mercury thermometer with muslin cap was increased above its theoretical value by stem heat conduction while that of the electrical resistance thermometer having 100 millimetres of tubular wick was not affected in this way. This is confirmed in Part II and the increase in the psychrometer coefficient of the mercury thermometer is calculated for a range of ventilation rates. The result is compared with experiment. The effects of modifications to the wick arrangements of the mercury and resistance thermometers are also calculated in order to show how the psychrometer coefficients could be made more consistent with the *Hygrometric Tables.*

PART I. LABORATORY INVESTIGATION OF PSYCHROMETER COEFFICIENTS OF WET-BULB THERMOMETERS USED IN THE METEOROLOGICAL OFFICE LARGE THERMOMETER SCREEN

1. INTRODUCTION

The relevance of the psychrometer coefficient used for thermometers exposed in the Meteorological Office large thermometer screen and published in the relevant *Hygrometric Tables* (Meteorological Office, 1964b) was first questioned in this country by J. MacDowall (1956). He deduced, from comparisons with Assman psychrometers at several sites, that the large screen was often underventilated. On the other hand, later comparisons at other sites by Sparks (private communication) produced the contrary result that the large screen was, on the whole, overventilated. The Assman psychrometer is no longer regarded as a satisfactory reference device in that the presence of the observer can introduce positive errors in both temperature and dew-point (Yoshitake and Shimizu, 1965). In a more recent paper, Clarkson (1971) reported evidence from two synoptic stations (Brize Norton and Little Rissington) showing that the wet-bulb depression derived from platinum resistance thermometers with four-inch-long stems exceeded that from mercury thermometers exposed in the same large screen by about 5 per cent, averaged over all the weather conditions encountered.

Painter (1973) has shown from comparisons with a specially designed aspirated psychrometer (Painter, 1970) at Kew that on average the mercury thermometer wet-bulb depression was 7 per cent too low and the resistance thermometer depression about 2 per cent too high averaged over all the weather conditions encountered. These results were obtained using a psychrometer coefficient of 0.799×10^{-3} K^{-1} for the naturally ventilated thermometers and 0.667×10^{-3} K^{-1} for the aspirated thermometers. These are the values published in the relevant Meteorological Office *Hygrometric Tables* (1964a, 1964b). The availability of an automatic dew-point hygrometer and a reference humidity calibration apparatus (Folland and Sparks, 1976) has made it possible to estimate the appropriate psychrometer 'constant' which should be applied to each of the wet-bulb thermometer systems for various ventilation rates.

2. HISTORICAL SURVEY

The source of the psychrometer constants used by the Meteorological Office is no longer published in the *Hygrometric Tables*. However, old editions clearly state that the constants (A) are those adopted by J. Pernter of the Austrian Weather Service. These are published in *Jelineks Psychrometer-Tafeln* (Pernter, 1903). Three values are quoted:

(a) $A = 1.2 \times 10^{-3}$ K^{-1} for calm conditions,

(b) $A = 0.8 \times 10^{-3}$ K^{-1} for slightly disturbed air ventilation rate 1 to 1.5 m s^{-1},

(c) $A = 0.656 \times 10^{-3}$ K^{-1} for very disturbed air ventilation rate 2.5 m s^{-1}.

The hygrometric tables for naturally ventilated thermometers are based on the second value, which assumes a ventilation rate of 1.0 to 1.5 m s^{-1}. The type of screen for which this ventilation rate is appropriate is not specifically stated, neither is the exact kind of thermometer. However, the psychrometer constant for slightly disturbed air is based on work by Regnault (1845). He investigated the behaviour both of cylindrical and of spherical wet bulbs and came to the conclusion that there was little to choose between them for the ranges of wet-bulb sizes he considered. From the psychrometric coefficients quoted, it is likely that he had considered the problem of heat conduction down the thermometer stem.

The standard mercury-in-glass wet-bulb thermometer used by the Meteorological Office has a spherical bulb 1.0 cm in diameter covered by a muslin cap which extends no more than 5 mm up the neck joining the stem to the bulb. Moreover, the wick in this area is in the form of a frill which does not adhere closely to the neck. It must accordingly be considerably less efficient in minimizing heat-conduction effects than a close-fitting wick would be. On the other hand, the resistance thermometer has a tightly fitting tubular wick which extends about 7.5 cm up the stem beyond the temperature-sensitive area, a total distance of 10 cm. The diameter of the cylindrical resistance bulb with wick is about 0.8 cm.

Kondo (1967) obtained expressions for the psychrometer coefficients for different sizes and shapes of wet bulb from the equations of the heat budget at the wick surface. He assumed no heat conduction down the stem. From his results it can be shown that the psychrometer coefficients of a spherical bulb 1.2 cm in diameter (equivalent to the mercury wet bulb including wick) and a cylindrical bulb 0.8 cm in diameter, both transversely aspirated, should be closely similar at all ventilation rates. If anything, the spherical

TABLE I. *Values of the psychrometer coefficient A from Kondo's theory compared to experimental values*

	Spherical bulb 1.2 cm in diameter		Resistance bulb 0.8 cm in diameter	
Ventilation rate	From Kondo's theory	Experimental value	From Kondo's theory	Experimental value
$m\ s^{-1}$	K^{-1}	K^{-1}	K^{-1}	K^{-1}
	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$
0	0.90	1.31	0.96	1.01
0.3	0.73	1.10	0.77	0.80
1.0	0.68	0.88	0.71	0.72
2.0	0.66	0.83	0.68	0.67
3.0	0.65	0.81	0.66	0.66
5.0	0.63	0.79	0.65	0.66

Figures based on Kondo's theory are calculated assuming mean wet- and dry-bulb temperature to be 18°C; mean wet-bulb depression 4.5°C.

bulb has a slightly lower psychrometer coefficient (Table I). If stem-conduction effects are absent, the mercury wet-bulb depression should thus be at least as great as that of the resistance thermometer for given humidity conditions. Evidence from the studies of both Clarkson and Painter clearly shows that the spherical muslin-capped mercury bulb must, in practice, have an appreciably larger coefficient than the cylindrical resistance bulb. Peers (private communication) had earlier shown that, at high ventilation rates, the resistance thermometer behaves as if it had no significant heat conduction down the stem. Referring to the psychrometer coefficients listed in Table I, it can be seen that the psychrometer coefficient of the resistance bulb is unlikely to be as high as $0.799 \times 10^{-3}\ K^{-1}$ for a wind speed of 1.0 to 1.5 m s^{-1}, while that for the mercury-in-glass thermometer at the same wind speed would only reach this value if the stem heat conduction assumed significant proportions. The good agreement between Kondo's theory and the experimental values (see later) for the resistance-bulb coefficient should be noted. Kondo's theory also shows, in agreement with many other workers, that variations in wet- and dry-bulb temperatures in the range of interest in this paper have no significant effect on the psychrometer coefficient for ordinary station purposes. Variations in atmospheric pressure for stations near sea level can also be neglected. In order to gain a better understanding of the underlying causes of this anomaly, and to demonstrate the reality of the experimental results reported by Painter and others, it was decided to carry out a wind-tunnel investigation.

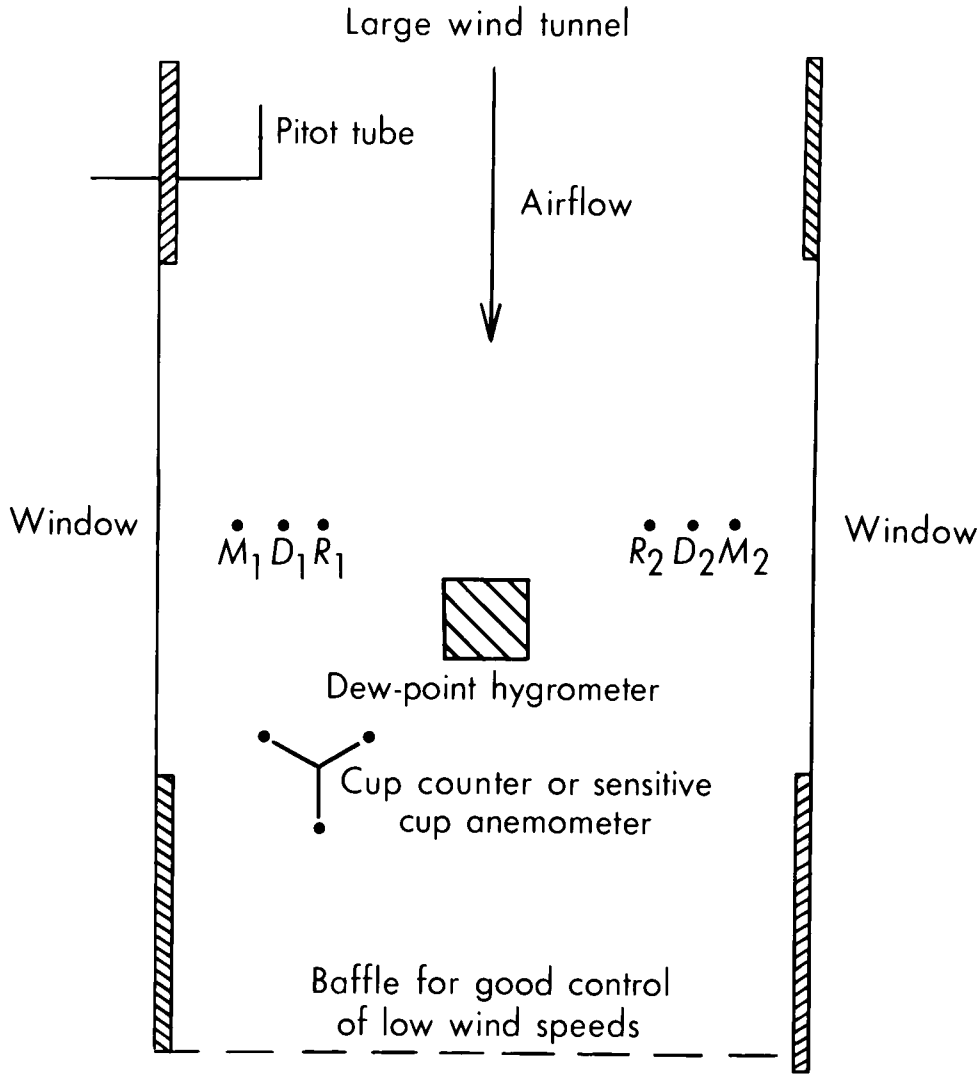

FIGURE 1. Experimental arrangement in the wind tunnel
M_1, M_2 Mercury wet-bulb thermometers,
D_1, D_2 Inspector's dry-bulb thermometers,
R_1, R_2 Resistance wet-bulb thermometers.

3. EXPERIMENTAL ARRANGEMENT

Figure 1 shows the general experimental layout. The thermometers were set up in line perpendicular to the wind direction in the wind tunnel and at such a distance apart that no thermometer would be likely to affect its neighbour. The mercury-in-glass and inspector's thermometers were read by eye through the tunnel window. A cup-counter anemometer was calibrated against the wind-tunnel pitot tube and placed downwind of the thermometers. A National Physical Laboratory-calibrated sensitive cup anemometer was used at speeds below 3 m s^{-1}. The cups were approximately at the thermometer bulb level and

care was taken to choose a position for the anemometer representative of the airflow past the thermometer bulbs. A large baffle was used downwind of the experimental volume to obtain a constant and easily controllable flow. The dew-point hygrometer used was designed and built in the Meteorological Office. The temperatures of the resistance thermometers and of the dew-point hygrometer mirror were measured with the aid of a calibrated Wheatstone Bridge. Several readings of the mercury thermometers were made in quick succession to reduce the effects of parallax and observer error. The wind speed was varied in the range 0 to 9 m s^{-1} and at least 20 minutes were allowed for the thermometers to settle down between changes of wind speed. Subsidiary experiments indicated that spatial variations of dew-point could be neglected in the experiments and that changes of dew-point with time were slow. During the experiments the resistance-thermometer wicks were shortened to see if stem heat conduction was sufficient to alter the psychrometer coefficient. In another experiment a length of aircraft-psychrometer tubular wick, 1.4 cm in diameter and 5 cm long, was placed over the lower stem of one of the mercury thermometers and tied to the thermometer's neck. A muslin cap was placed over the bulb so that the cap and tubular wick overlapped, the two being held tightly together by thread. The dew-point hygrometer was compared with a precision humidity generator before and after the series of experiments and small corrections applied (Folland and Sparks, 1976). The mercury-in-glass thermometer and resistance thermometers were calibrated against an inspector's thermometer.

4. RESULTS OF WIND-TUNNEL TESTS

Figure 2 illustrates the variation of A with ventilation rate for the resistance thermometer for wicks 5.0 cm and 10.0 cm long, and for the mercury thermometer with muslin cap with and without the extra length of tubular wick. Each point on the graph is the mean of coefficients obtained for the two thermometers exposed in the tunnel (see Figure 1). Referring to Table I, it is seen that the experimental values of A for the resistance bulb with 10 centimetres of wick agree well with Kondo's predictions.

The psychrometer coefficients of the two commonly used wet bulbs (curves A and B) are appreciably different at all ventilation rates. At 1.25 m s^{-1}, the mean value of the commonly accepted screen ventilation rate, $A = 0.87 \times 10^{-3}$ K^{-1} for the wet bulb with muslin cap and 0.70×10^{-3} K^{-1} for the resistance bulb. Both values are appreciably different from the value of 0.799×10^{-3} K^{-1} assumed in the tables, and would give rise to a mercury thermometer wet-bulb depression for average conditions encountered in the United Kingdom apparently 4 per cent too small, and a resistance thermometer wet-bulb depression apparently 6 per cent too large. The present results and those of Painter can be reconciled if the mean ventilation rate in the large screen at Kew during this comparison was 0.6 m s^{-1}. Figure 2 shows that for this ventilation rate A is 0.76×10^{-3} K^{-1} (resistance) and 0.93×10^{-3} K^{-1} (mercury) corresponding to wet-bulb depression 2 per cent too large (resistance) and 7 per cent too small (mercury). The mean dry-bulb temperatures in Painter's work were only a few degrees lower than those encountered in the wind tunnel.

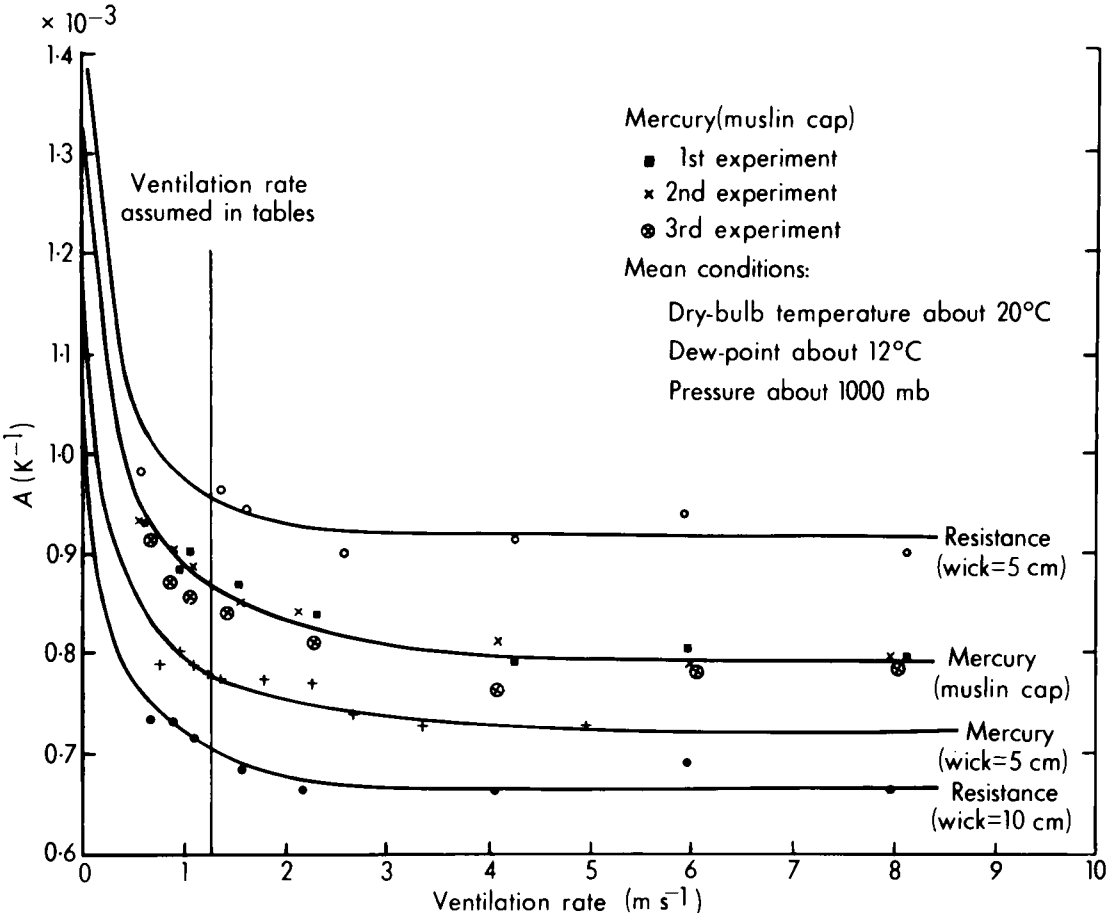

FIGURE 2. Variation of psychrometer coefficients with ventilation rate

The estimate of the mean ventilation rate at Kew is in remarkable agreement with an estimate by Bultot and Dupriez (1971) who arrived at the same figure using hot-wire anemometers in a large screen at Uccle. Thus the hitherto accepted value for the mean ventilation rate in a large screen must be regarded as suspect, at least at stations with exposures and wind regimes similar to those at Kew and Uccle. It is interesting to compare these results with those for the mercury thermometer with the extra piece of tubular wick. Figure 2 shows that A is now close to its assumed value at both 0.6 m s^{-1} and 1.25 m s^{-1}, being 0.83 x 10^{-3} K^{-1} and 0.77 x 10^{-3} K^{-1} respectively. Although a long piece of tubular wick had been added there was probably residual heat conduction as the wick extended along the outer sheath of the double-walled thermometer and was not in contact with the thermometer stem itself. This accounts for the fact that the psychrometer coefficient is greater than the calculated values for the spherical wet-bulb in Table I.

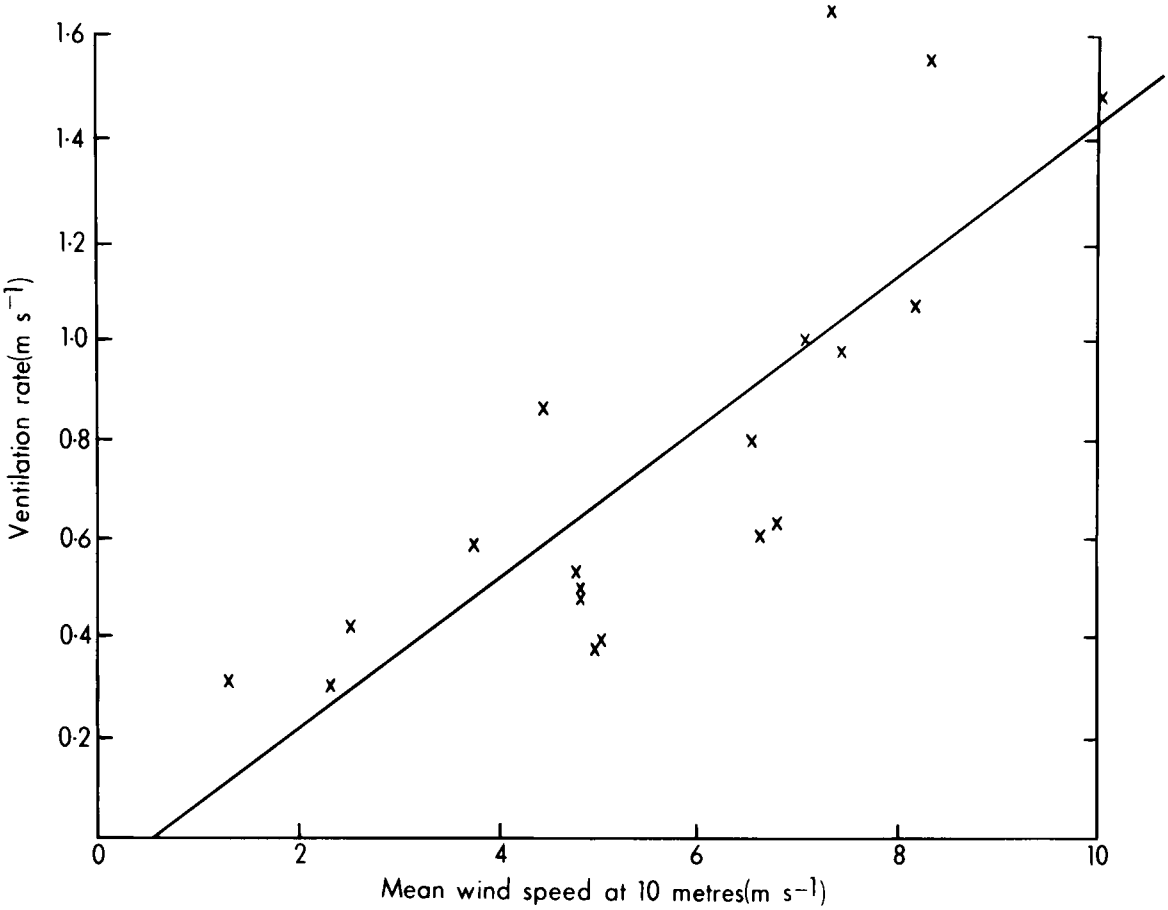

FIGURE 3. Relation between ventilation rate in screen and wind speed at 10 metres
$r = 0.81; y = -0.08 + 0.15x$; 95 per cent confidence limits on slope 0.15 ± 0.04.

5. FIELD INVESTIGATION OF THE VENTILATION RATE IN A LARGE SCREEN

A trial was carried out at a fairly well exposed site at Bracknell between September 1971 and April 1972 to investigate the relationship between the ventilation rate in the screen, the ambient wind speed at the same level (1.25 metres) and the wind speed at the standard height of 10 metres. No thermograph or hygrograph was placed in the screen and the screen door was kept shut. A standard Meteorological Office anemograph was used for the readings at 10 metres while a special 'Porton' type anemometer of lower starting speed was used at screen level (1.25 metres above the ground). A very sensitive miniature anemometer with scoop-shaped cups as described by Sheppard, Tribble and Garrett (1972) was used inside the screen near the positions of the thermometer bulbs. A total of 19 successful runs giving simultaneous ten-minute mean wind speeds was obtained. Figure 3 shows the relationship between the ventilation rate in the screen and the wind speed at 10 metres. Over the range of 10-metre mean wind speed in the experiments the data are well fitted with a straight line passing near the origin which suggests that in this range

the mean screen ventilation rate is about 15 per cent of the mean wind speed at 10 metres. It did, however, vary from between 6 and 25 per cent of the 10-metre mean wind speed with parallel variations in the ratio of the 1.25-metre wind speed to that at 10-metres. For a mean wind speed at 10 metres of 5 m s^{-1} the corresponding mean ventilation rate in the screen is only about 0.75 m s^{-1}. The speed of 5 m s^{-1} is close to the average yearly mean 10-metre wind speed over the British Isles, though 4 m s^{-1} would be more representative of Kew. Thus the ventilation rate in the large screen is little more than half that hitherto assumed. The Belgian screen used at Uccle was probably very similar to that at Bracknell. Thus Bultot and Dupriez's mean ventilation rate of 0.6 m s^{-1} would be consistent with a mean 10-metre wind speed of about 4 m s^{-1}.

6. THE UNSATISFACTORY NATURE OF THE MERCURY (MUSLIN-CAP) WET BULB

A mathematical theory has been developed which confirms that the increase of the psychrometer coefficient of the mercury thermometer over that expected is a result of stem heat conduction. This is described in Part II of this paper which also discusses heat conduction effects down the resistance bulb (negligible with 10 cm of wick present). The inherently variable performance of the present design of mercury wet bulb will be pointed out here. Figure 4 shows how the psychrometer coefficient varies with effective length of wick up the stem of the thermometer for different aspiration rates. In practice effective lengths from 1.5 to 5 mm have recently been observed at different synoptic and climatological stations in Great Britain. This is partly the result of the use of two different

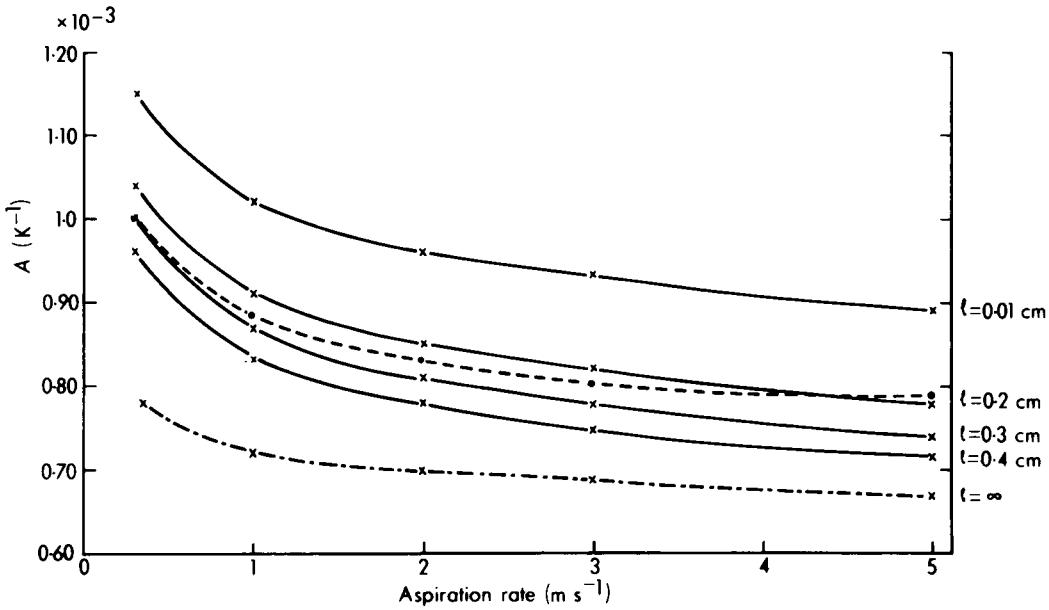

FIGURE 4. Predicted psychrometer coefficient – muslin-cap thermometer, $\overline{T + T_w} = 18°C$

Solid lines denote predicted values, pecked lines denote experimental values and dots and dashes denote values based on column 4 of Table IV in Part II (no stem heat conduction).

types of muslin wick – muslin 'cap' and muslin 'strip'. The dotted line represents the values of A found in the wind tunnel experiments. The possible variation in the mean value of A (the mean value is well represented by the wind-tunnel results) is about ± 15 per cent at an aspiration rate of 0.6 m s^{-1} or typically ($\overline{T + T_w}$ = 18°C) ± 7 per cent in $T - T_w$. Thus some existing mercury wet bulbs using muslin strip might behave almost identically to resistance bulbs with 10 cm of wick, while others, particularly poor bulbs using muslin cap, could show even greater disagreement than the experiments in this paper indicate. It is thus essential that the inherent variability in the behaviour of the present mercury-in-glass thermometer wet-bulb arrangement be removed.

7. PRACTICAL SIGNIFICANCE OF THE RESULTS

The wet-bulb depression obtained from a naturally ventilated psychrometer will often be in error whatever 'constant' is employed. The constant is a coefficient, varying very markedly, mainly with ventilation rate, except when the wet bulb has a diameter much smaller than is being considered here. In calm conditions (Figure 2) the resistance thermometer coefficient is 1.0 x 10^{-3} K^{-1} while the coefficient of the mercury thermometer with muslin cap is 1.31 x 10^{-3} K^{-1}. At 1.0 m s^{-1} the coefficients are 0.72 x 10^{-3} K^{-1} and 0.88 x 10^{-3} K^{-1} respectively. Thus natural variation of the coefficient with the ventilation rate is considerably greater than are differences in its values between different types of thermometer. Kondo's theory also indicates that the coefficient becomes appreciably more sensitive to variations in wet-bulb depression and mean wet- and dry-bulb temperature in near calm conditions (less than 0.2 m s^{-1}). These are the conditions where natural rather than forced convection predominates. Three effects will be considered in detail.

7.1. *Expected differences in indicated wet-bulb depression between the resistance and mercury (muslin-cap) thermometers*

Figure 5 shows the values found in the wind tunnel. The average extra wet-bulb depression of the resistance thermometer over the mercury thermometer with muslin cap is about 10 per cent for the likely range of ventilation rates. In the Appendix (Section 2) the expression relating the wet-bulb depression to the psychrometer coefficient is derived to be very nearly:

$$\Delta (T - T_w) = \frac{(U - 1)e_D}{A + \overline{\beta}_w} \quad \ldots (1)$$

where U is the true relative humidity expressed as a fraction, e_D is the saturation vapour pressure over a plane water surface at the dry-bulb temperature and A is the psychrometer coefficient expressed in mb K^{-1}, $\overline{\beta}_w$ is the rate of saturation vapour pressure e_s at the mean of the wet- and dry-bulb temperatures, $\overline{T + T_w}$; the relationship between these last is approximately

$$\overline{\beta}_w = \frac{de_s}{d(\overline{T + T_w})}. \quad \ldots (2)$$

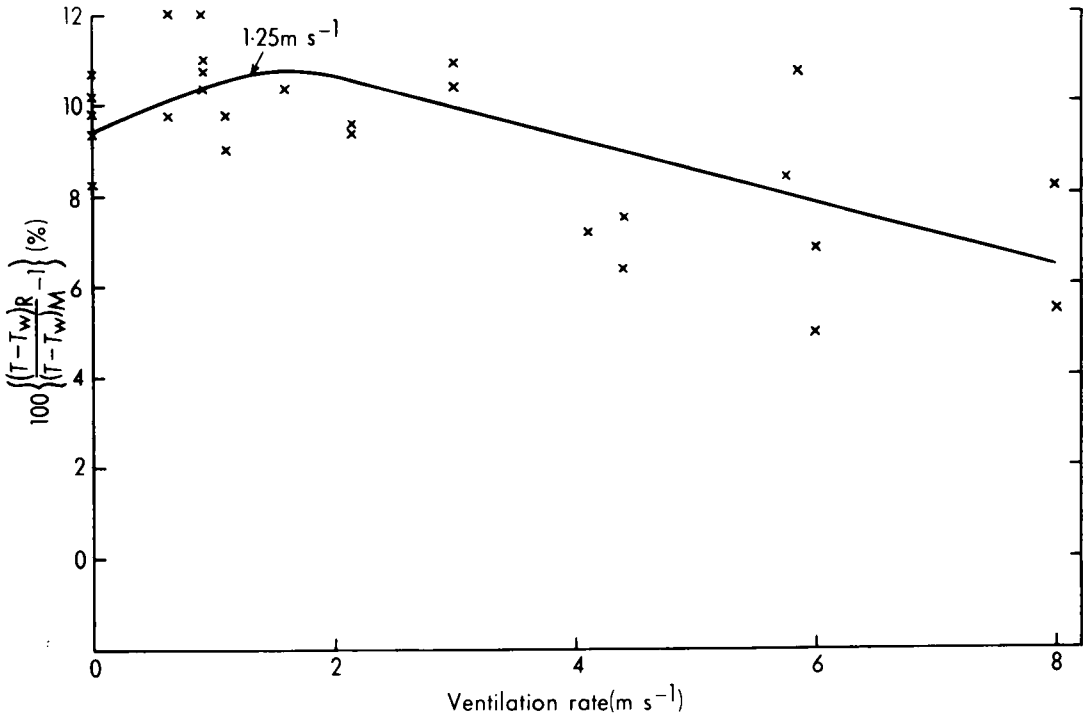

FIGURE 5. Comparison of wet-bulb depressions of mercury bulb (muslin cap) and resistance bulb with 10-cm wick as a function of ventilation rate $\overline{T + T_w}$ = 18°C and $T - T_w$ about 5°C

$(T - T_w)_R$ is wet-bulb depression of resistance thermometer pair;
$(T - T_w)_M$ is wet-bulb depression of mercury thermometer pair.

When two thermometers with different psychrometer coefficients are considered, for the same U and e_D there is

$$\frac{\Delta(\overline{T+T_w})_2}{\Delta(\overline{T+T_w})_1} = \frac{A_1 + \bar{\beta}_{w,1}}{A_2 + \bar{\beta}_{w,2}} . \qquad \ldots(3)$$

Now unless the coefficients are very different it can be said that, for two thermometers exposed to the same conditions, $\bar{\beta}_{w,1} = \bar{\beta}_{w,2}$.

Then

$$\frac{\Delta(\overline{T+T_w})_2}{\Delta(\overline{T+T_w})_1} = \frac{A_1 + \bar{\beta}_w}{A_2 + \bar{\beta}_w} \qquad \ldots(4)$$

where $\bar{\beta}_w$ is evaluated from the mean wet- and dry-bulb temperatures of the two thermometers. Since A_1 and A_2 are virtually invariant with wet-bulb temperature, the maximum value of $\Delta(\overline{T+T_w})_2 / \Delta(\overline{T+T_w})_1$ occurs when $\bar{\beta}_w$ is least, i.e. for low values of $\overline{T+T_w}$.

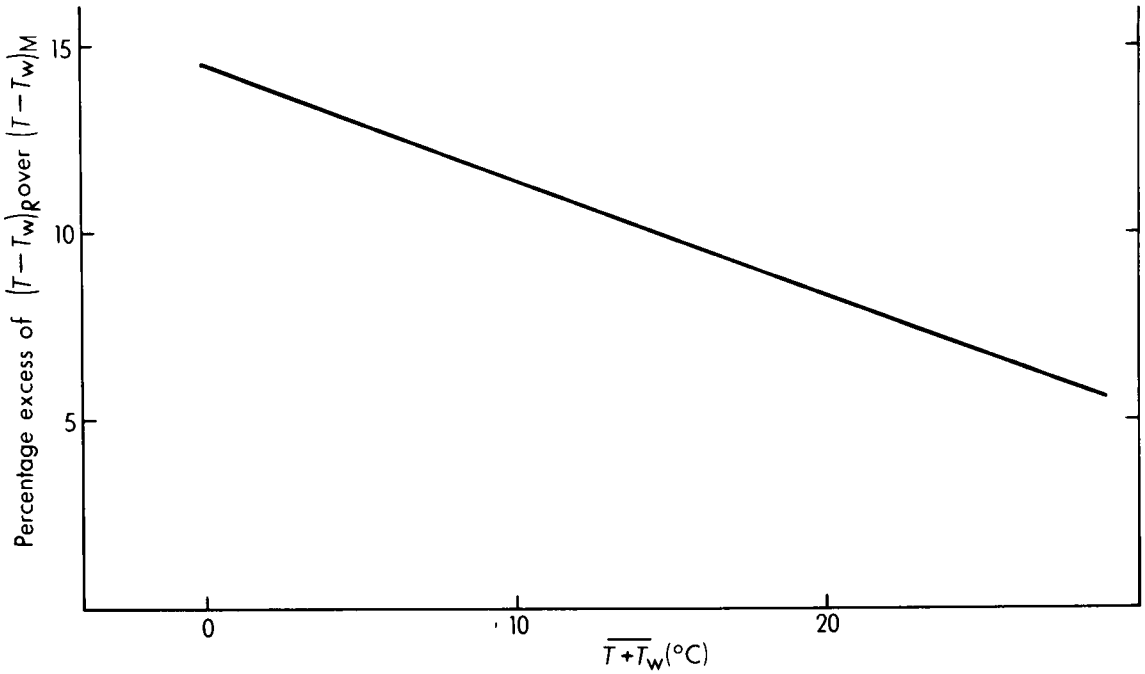

FIGURE 6. Percentage difference in wet-bulb depression as a function of $\overline{T + T_w}$ from theory for moderate values of $T - T_w$.

The variation of $\overline{\beta}_w$ for temperatures above 0°C is tabulated by Wylie (1968b). $\overline{\beta}_w$ varies exponentially from about 0.45 at $\overline{T + T_w}$ = 0°C to 2.45 at $\overline{T + T_w}$ = 30°C. Figure 6 shows the expected variation of the difference in wet-bulb depression, $(T - T_w)$, for the resistance thermometer with a 10-cm wick, and mercury-in-glass thermometer with muslin cap between $\overline{T + T_w}$ = 0°C and $\overline{T + T_w}$ = 27°C for (A_1/A_2) = 1.23. This value of A_1/A_2 is applicable, for practical purposes, over all likely ventilation rates. Thus the observed difference will depend on $\overline{T + T_w}$, being least at high temperatures and greatest at low temperatures. Trials held under different conditions of mean temperature would not therefore be expected to show the same difference in wet-bulb depression even if the respective psychrometer coefficients did not change.

Various other investigators have commented on such differences. In 1968, in a trial of the Meteorological Office Weather Observing System Mk 1, an excess of 7 per cent was found in the mean wet-bulb depression of a resistance thermometer over that of a mercury wet bulb. In the Appendix (Section 1) the relationship between absolute errors in A and the resulting errors in wet-bulb depression is discussed.

7.2. *Errors in calculated dew-points and relative humidities*

Figures 7 and 8 illustrate the errors of the mercury-in-glass and resistance thermometer wet-bulb estimates of dew-point and relative humidity respectively as a function of ventilation rate for $\overline{T + T_w}$ = 18°C when the *Hygrometric Tables* are used. Errors at lower temperatures would be slightly larger and vice versa.

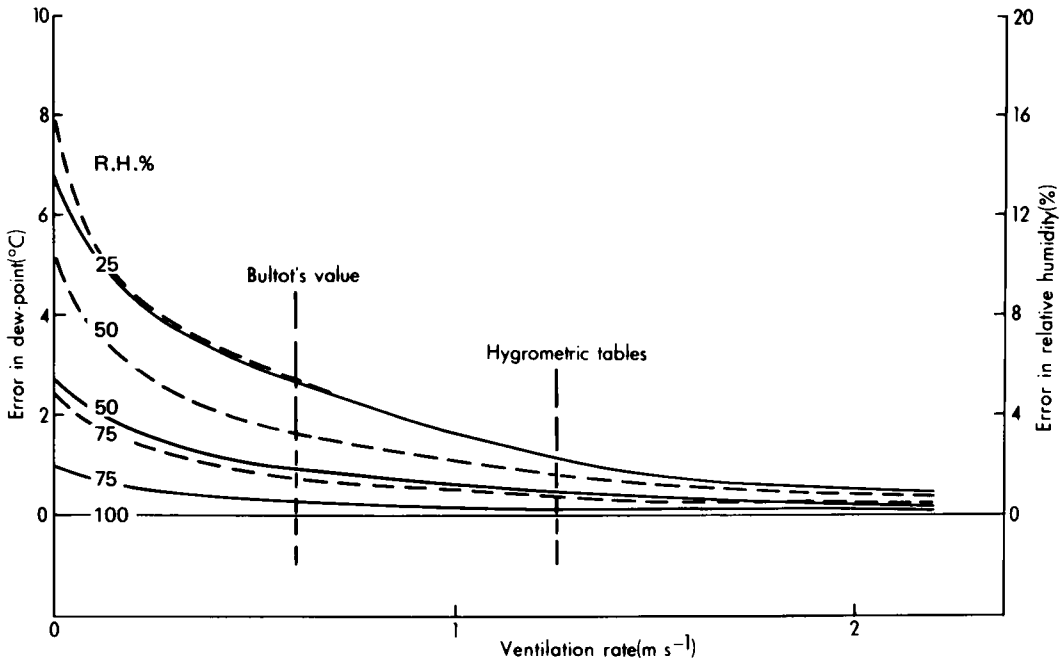

FIGURE 7. Expected errors in the mercury (muslin-cap) thermometer psychrometer
Solid lines show dew-point; broken lines show relative humidity.
$\overline{T + T_w} = 18°C$.

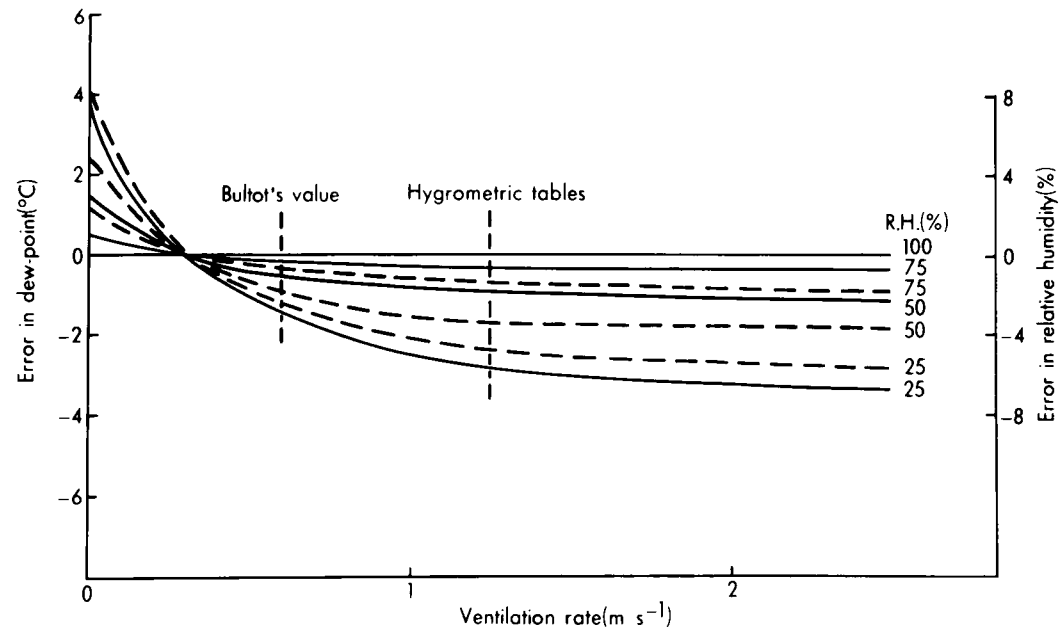

FIGURE 8. Expected errors in the resistance thermometer psychrometer
Solid lines show dew-point; dashed lines show relative humidity.
$\overline{T + T_w} = 18°C$.

Figure 7 indicates that the mercury thermometer psychrometer with muslin cap always overestimates dew-point and relative humidity except in very strong winds when the correct values would be indicated. As a result it is likely that the errors of the mercury thermometer with muslin cap in light winds are likely to be greater than have previously been expected. On the other hand Figure 8 shows that the resistance thermometer psychrometer gives correct values at a ventilation rate of 0.3 m s^{-1} and underestimates dew-point and relative humidity at higher ventilation rates.

Zobel (1965) studied errors caused by underventilation at Farnborough in hot weather in July 1959, using a hair hygrograph as a reference. Apparent errors of up to 13 per cent relative humidity were noted but he ascribed some of the differences to an error in the hygrograph at low humidities which caused the hygrograph to underestimate relative humidity. This conclusion was based upon an observation that the hair hygrograph read 5 per cent low relative to the relative humidity derived from the wet-bulb depression at about 25 per cent relative humidity and a temperature of 31.7°C. He assumed that a 10-metre wind speed of 3.5 m s^{-1} would give rise to the mean ventilation rate in the screen assumed in the *Hygrometric Tables* (1.0 to 1.5 m s^{-1}). If the ventilation rate was nearer the 0.6 m s^{-1} obtained by Bultot, the overestimation of the wet- and dry-bulb indication would be 4 per cent relative humidity and the hygrograph error would be 1 per cent rather than 5 per cent. The 'apparent errors' quoted by Zobel can thus be accepted as most likely being nearer the truth than his 'corrected errors' and hence the effects of

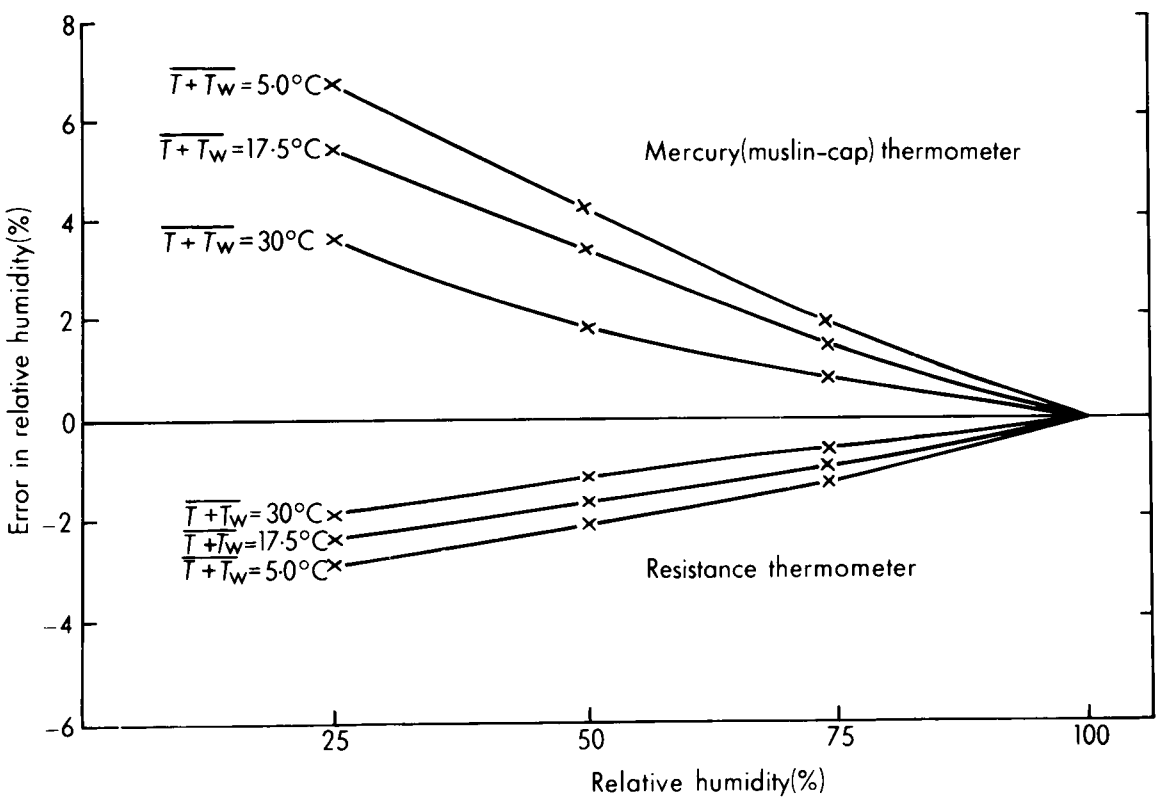

FIGURE 9. Expected mean errors in relative humidity as a function of relative humidity at a mean ventilation rate of 0.6 m s^{-1}.

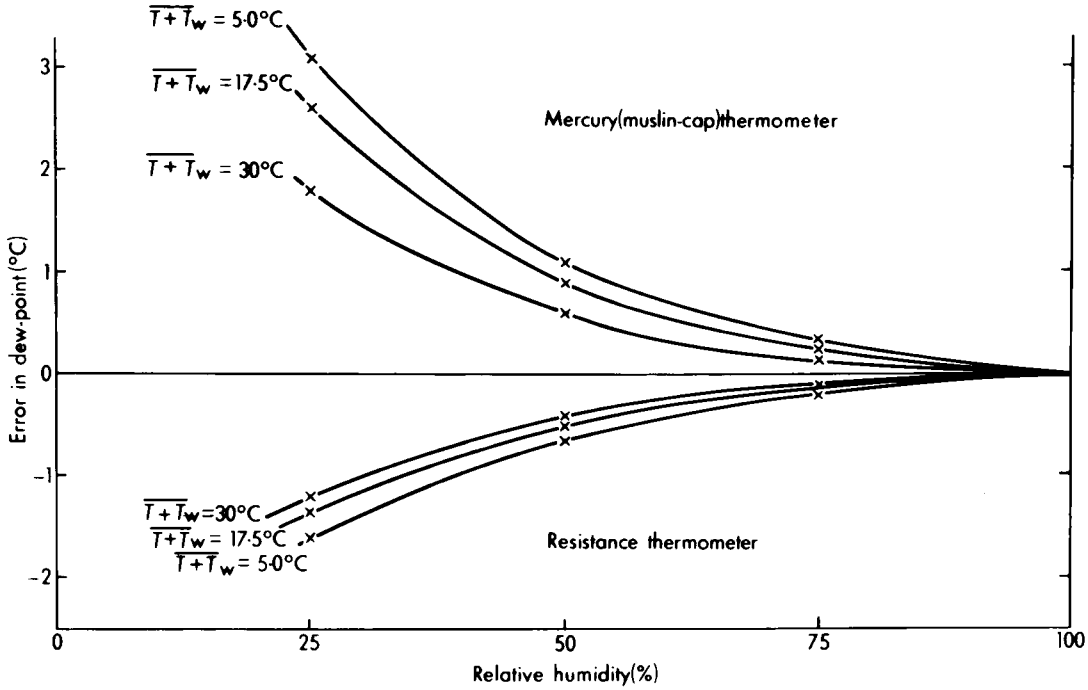

FIGURE 10. Expected mean errors in dew-points as a function of relative humidity at a mean ventilation rate of 0.6 m s^{-1}.

underventilation worse than he deduced. It is unsatisfactory that the resistance and mercury thermometer wet bulbs should often give markedly different relative humidities and dew-points. Some forecasting criteria may be significantly affected, for instance potato-blight forecasting, when high relative humidity criteria were based on observations with mercury-in-glass thermometers while some of the forecasts were based on resistance thermometers.

7.3 *Mean climatological differences in indicated relative humidity between the mercury and resistance thermometers and the true errors of both*

Figures 9 and 10 illustrate the mean likely climatological errors of dew-point and relative humidity as functions of dry-bulb temperature and relative humidity, assuming a mean ventilation rate of 0.6 m s^{-1} in the screen, when the *Hygrometric Tables* are used. The difference between the thermometers and the errors in the indicated relative humidity increases as relative humidity decreases. In typical mean summer conditions in southern England (e.g. dry bulb 18°C, relative humidity 65 per cent) the difference in indicated relative humidity is 3.5 per cent, the mercury thermometer overestimating the relative humidity by over 2 per cent and the resistance thermometer underestimating the relative humidity by over 1 per cent. Because the muslin-capped mercury-thermometer wet-bulb body is so ill defined, significant variations in its efficiency must occur. It is worth noting that the narrower the wet-bulb cylinder the less sensitive it is to variations of wind speed. A fine wire resistance or thermocouple thermometer with a diameter of 1 mm including

wick would have a psychrometer coefficient almost invariant with ventilation rate for practical purposes. It would have the further advantage that it would behave more nearly as an ideal sensor of temperature and humidity fluctuations which can be integrated electronically over any desired integration period.

8. COMPARISONS OF THE OBSERVED ERRORS IN DEW-POINT WITH THOSE EXPECTED FROM THE WIND-TUNNEL EXPERIMENTS

Painter's 1970–71 comparisons of a series of mercury-in-glass muslin-cap wet bulbs and resistance bulbs against a Kew-pattern aspirated psychrometer have been reinterpreted in terms of the errors observed in dew-point for the naturally ventilated bulbs and compared with predictions based on the results of the wind-tunnel experiments. Table II shows the comparisons of observed and expected dew-point errors for the muslin-cap and resistance thermometers. The agreement is good for the particular conditions of relative humidity, mean wet- and dry-bulb temperature and ventilation rate observed. This also suggests that the ventilation-rate experiment performed at Bracknell is applicable to the Kew exposure.

Experiments have been carried out at Kew Observatory to determine the mean wick lengths needed to give agreement averaged over a representative set of weather conditions at Kew with dew-points calculated from the Kew-pattern aspirated psychrometer when a value of the psychrometer coefficient of 0.799×10^{-3} K^{-1} is used for naturally ventilated bulbs. For the mercury thermometer, some extension of the wick up the thermometer stem is needed if the thermometer is to be made into a repeatable wet bulb. Further variations of wick length up the sheathed stem have only a small effect. About 1.5 cm of wick seems to give the best result at Kew. The resistance-thermometer wick can be adjusted to any length quite easily.

Table III summarizes the results compared to predictions based on theory for different ventilation rates. A resistance thermometer wick length of about 8 cm would seem satisfactory for average conditions at Kew.

TABLE II. *Errors in wet-bulb thermometers – comparison of predicted with actual errors in dew-point*

Wet-bulb thermometer type	Range of relative humidity (per cent)		
	100–75	74–50	49–30
Mercury (muslin cap)	Actual 0.3 ± 0.1 n = 23 Predicted 0.2	Actual 0.7 ± 0.1 n = 80 Predicted 0.6	Actual 0.9 ± 0.1 n = 30 Predicted 1.1
Resistance	Actual −0.2 ± 0.1 n = 23 Predicted −0.1	Actual −0.2 ± 0.1 n = 80 Predicted −0.3	Actual −0.35 ± 0.2 n = 30 Predicted −0.6

Winds in the range 0–9 m s^{-1} at 10 m; predicted figures for actual conditions of relative humidity, wind etc. occurring. Limits quoted are twice the standard error of the mean. n denotes number of occasions.

TABLE III. *Resistance thermometer wet bulb – theoretical and observed lengths of wick corresponding to psychrometer coefficient* $A = 0.80 \times 10^{-3}\ K^{-1}$ *for various mean screen-ventilation rates*

Wind at 10 m	Theoretical wick length needed	Observations at Kew
$m\ s^{-1}$	*cm*	*cm*
0–2.5	Impossible	Impossible
2.5–5	8–10	8–10
5–7.5	7–8	8
7.5–10	6.6–7	–

An addition of 3 mm has been allowed in theoretical estimates for typical frayed top edges of the wick.

9. CONCLUSIONS

In this laboratory investigation of psychrometer coefficients of wet-bulb thermometers used in the Meteorological Office large thermometer screen the following conclusions were reached.

(a) Except at high humidities, the temperature of a muslin-capped mercury wet-bulb thermometer and a resistance thermometer with a 10-cm wick exposed in a large thermometer screen differ markedly, the difference being greatest at low relative humidities.

(b) By fitting suitable wick lengths the two types of wet-bulb thermometer can be made to agree with one another.

(c) With wick lengths chosen so that both wet-bulb thermometers read the same temperature, errors of relative humidity and dew-points derived by means of any single value of the psychrometer coefficient depend on the naturally occurring ventilation rate within the screen.

(d) Wick lengths can be selected so that errors in relative humidity and dew-point derived from the present hygrometric tables or humidity slide rule are a minimum for commonly occurring ventilation rates such as obtain with the large thermometer screens at Kew, Uccle and Bracknell.

(e) If in future either the resistance or mercury-in-glass thermometers is changed for one of a new design, sufficient of the experiments described in this paper should be repeated to determine the best wick configuration so that indicated humidities are obtained as accurately as possible from the use of existing tables, or if deemed appropriate, from the use of revised tables.

PART II. THEORETICAL INVESTIGATION OF STEM HEAT CONDUCTION OF WET-BULBS USED IN THE METEOROLOGICAL OFFICE LARGE THERMOMETER SCREEN

10. INTRODUCTION

An attempt is made to calculate the effect of stem heat conduction in mercury-in-glass and platinum resistance thermometers to assess its contribution to the systematic errors quoted in Part 1 theoretically. The problem is not soluble exactly as the mercury-in-glass wet-bulb system is ill defined and the details of the internal construction of the resistance thermometer are complex. The results to be presented show, however, that a realistic estimate can be made of the stem heat conduction in both cases, which satisfactorily accounts for the observed psychrometer coefficients when the problem of ventilation is explicitly allowed for as well. Part 1 contained a description of the experiment on the psychrometer coefficients and the practical significance of the results. These results will be quoted only where a comparison is being made with theoretical predictions.

11. STEM HEAT CONDUCTION IN THE MERCURY-IN-GLASS THERMOMETER WITH MUSLIN CAP

11.1. *General*

It is necessary first to obtain a reliable estimate of the psychrometer coefficient in the absence of stem heat conduction. Wylie (1968a) indicates how the various theoretical estimates so far made of the psychrometer coefficients of real thermometers underestimate their values when compared with experiment. The underestimation may amount, after allowance for radiative heat transfer, to as much as 9 per cent. Table IV displays values of the psychrometer coefficient of the mercury-in-glass thermometer as a function of venti-

TABLE IV. *The psychrometer coefficient of the mercury-in-glass thermometer (spherical bulb) radius 5.8 millimetres*

Ventilation rate	Experimental psychrometer coefficient	Kondo's estimate	Revised estimate	Experimental value minus revised estimates
$m\ s^{-1}$	A $mb\ K^{-1}$	A $mb\ K^{-1}$	A $mb\ K^{-1}$	dA $mb\ K^{-1}$
0	1.31	0.90		
0.3	1.00	0.73	0.78	0.22
1.0	0.88	0.68	0.72	0.16
2.0	0.83	0.66	0.70	0.13
3.0	0.81	0.65	0.69	0.12
5.0	0.79	0.63	0.67	0.11

lation rate, calculated from a theory by Kondo (1967). Most experiments, in the absence of stem heat conduction, show that the psychrometer coefficient of ordinary thermometer bulbs tends, at a sufficiently high ventilation rate, to within 2 per cent of 0.667 mb K^{-1}*. In Table IV the values in column 4 are Kondo's values increased by about 6 per cent to make the psychrometer coefficient of the thermometer 0.67 mb K^{-1} at a ventilation rate of 5 m s^{-1}. This can be regarded as a sufficiently accurate representation of the psychrometer coefficient as a function of ventilation rate (in the absence of stem heat conduction) for estimates of the increase in the coefficient due to stem heat conduction to be made. These estimates are shown in column 5 of Table IV.

In the following work it is assumed that, in the absence of stem heat conduction, the psychrometer coefficient does not vary with wet-bulb temperature. This is justified as the variation in Ferrel's equation of A with wet-bulb temperature (Marvin, 1941) is small. Kondo's work also confirms this. Figure 11 shows a simplified diagram of the mercury wet bulb. The real geometry is complex. The wick is generally ill fitting with folds over the bulb, making it non-circular, while the extension of the wick over the glass neck is in the form of a ragged frill. This only touches the lower part of the neck, there being an air gap between the upper part of the frill and the upper neck which varies around the neck circumference. In conditions of forced convection the aspiration rate of the bulb is fairly well defined but that of the upper neck much less so.

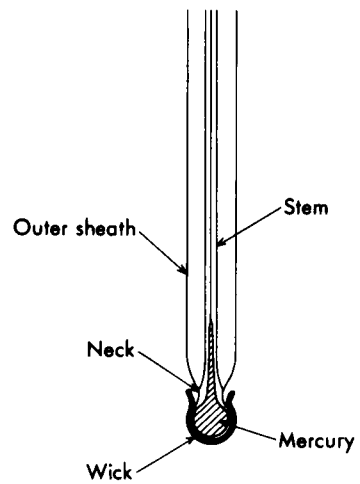

FIGURE 11. The mercury-in-glass thermometer

11.2. *Theoretical model*

It is assumed that the wet-bulb system consists of a spherical bulb attached to an infinite, uniform, glass rod with a mercury thread running through its centre. The wick is assumed to extend a certain length along the rod and to be in perfect contact with it. Figure 12 illustrates the model used in the calculations. An equation will now be derived for the increase in the psychrometer coefficient as a result of stem heat conduction.

* All the psychrometer coefficients quoted in Part II include the pressure term which is assumed to be 1000 mb.

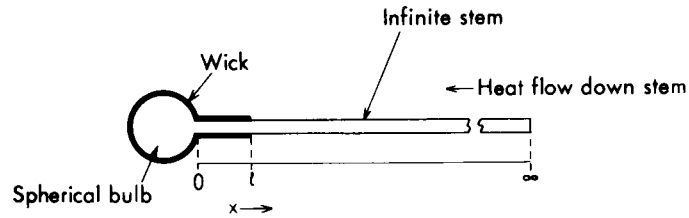

FIGURE 12. Model of the mercury-in-glass thermometer used in the theoretical calculations

The equilibrium heat transfer equation for the spherical bulb is obtained by equating the rate of loss of heat by evaporation to that gained by convection, radiation and stem conduction. Let h_v represent the vapour heat transfer coefficient, L the latent heat of evaporation of water, S the surface area of the sphere, e'_w the vapour pressure at the wet-bulb temperature of the sphere, e the vapour pressure of the ambient air, h_c the convective heat transfer coefficient, h_r the radiative heat transfer coefficient, T the ambient air temperature, T'_w the temperature of the wet-bulb surface of the sphere and (dQ/dt) the rate of flow of heat by conduction along the stem. Then for the wet-bulb sphere

$$(h_v)_{sph}\, LS(e'_w - e) = (h_c + h_r)_{sph}\, S(T - T'_w) + \left(\frac{dQ}{dt}\right)_{x=0} \qquad \ldots (5)$$

where suffix sph denotes the heat or vapour transfer coefficient for a sphere. This expression assumes the sphere to be at a uniform temperature. Now let e_w be the vapour pressure of the wet-bulb sphere in the absence of stem heat conduction and T_w the corresponding wet-bulb temperature. The psychrometer equation for this condition can then be written as

$$(h_v)_{sph}\, LS(e_w - e) = (h_c + h_r)_{sph}\, S(T - T_w)\,. \qquad \ldots (6)$$

Subtracting (6) from (5) gives

$$(h_v)_{sph}\, LS(e'_w - e_w) = (h_c + h_r)_{sph}\, S(T_w - T'_w) + \left(\frac{dQ}{dt}\right)_{x=0}\,. \qquad \ldots (7)$$

Consider a length, dx, of the wick-covered stem. The net rate of flow of heat, $(dQ/dt)'$, into such an element can be estimated by a familiar argument in heat-conduction theory which gives

$$\left(\frac{dQ}{dt}\right)' = kB\,\frac{\partial^2 T''_w}{\partial x^2}\,dx$$

where T''_w is the wet-bulb temperature at any part of the wick-covered stem, B is the mean cross-section of the wick-covered stem, and k is the effective thermal conductivity of the stem. Heat is also lost from the stem by evaporation and gained by convection and radiation. Let r be the radius of the stem and e''_w the vapour pressure at the wet-bulb temperature of the stem. Then the following is obtained from (6) for the element dx

$$2\pi r(h_v)_{cyl} L(e''_w - e_w)\,dx = 2\pi r(h_c + h_r)_{cyl}\,(T_w - T''_w)\,dx + Bk\,\frac{\partial^2}{\partial x^2}\,(T''_w - T_w)\,dx \qquad \ldots (8)$$

where cyl is the suffix denoting a cylinder.

Now, to a high degree of approximation

$$e''_w - e_w = \bar{\beta}_w (T''_w - T_w)$$

where $\bar{\beta}_w$ is the rate of change of vapour pressure with wet-bulb temperature at the mean value of T''_w and T_w. When this substitution is made and the equation is rearranged

$$\frac{\partial^2}{\partial x^2}(T''_w - T_w) = \frac{2}{kr}(h_v L\bar{\beta}_w + h_c + h_r)_{cyl}(T''_w - T_w) . \quad \ldots (9)$$

The solution of this differential equation is of the form

$T''_w - T_w = \{F_1 \exp(mx)\} + \{G_1 \exp(-mx)\}$ where F_1 and G_1 are constants and

$$m = \left\{\frac{2}{kr}(h_v L\bar{\beta}_w + h_c + h_r)_{cyl}\right\}^{1/2} .$$

The boundary conditions are

$$T''_w = T_s \text{ at } x = l,$$

$$T''_w = T_w \text{ at } x = 0,$$

where T_s is an unknown temperature at the boundary of the wick- and non-wick-covered parts of the stem.

When the approximation $T_w \approx T'_w$ is made, and solving for the values of the constants and differentiating,

$$\frac{d}{dx}(T''_w - T'_w) = \frac{m(T_s - T'_w)\cosh mx}{\sinh ml} ; \quad \ldots (10)$$

T_s must now be eliminated as an unknown. A similar equation to (9) can be derived from the non-wick-covered stem,

$$T' - T = F_2 \exp(m'x) + G_2 \exp(-m'x)$$

where

$$m' = \left[\frac{2}{k_d r_d}(h_c + h_r)'_{cyl}\right]^{1/2} .$$

A prime denotes the dry stem while k_d and r_d are the thermal conductivity and radius of the dry stem. The boundary conditions are

$$T' = T \text{ at } x = \infty ,$$

$$T' = T_s \text{ at } x = l .$$

Solving for the values of the constants and differentiating,

$$\frac{d}{dx}(T' - T) = m'(T - T_s)\exp\{-m'(x - l)\} . \quad \ldots (11)$$

Now the flux of heat at $x = l$ due to conduction must be the same in the non-wick-covered stem as in the wick-covered stem. Thus the fluxes of heat from (10) and (11) at $x = l$ can be equated

$$k_d r_d^2 m'(T - T_s) = \frac{kr^2 m(T_s - T'_w)}{\tanh ml} .$$

When this is rearranged we obtain

$$T_s - T'_w = \frac{T - T'_w}{1 + \epsilon} \qquad \ldots (12)$$

where

$$\epsilon = \frac{kmr^2}{k_d r_d^2 m' \tanh ml} .$$

Now the rate of ingress of heat by conduction into the wet-bulb sphere at $x = 0$ was given by the equation (from(8)):

$$\left(\frac{dQ}{dt}\right)_{x=0} = Bk \left[\frac{d}{dx}(T''_w - T'_w)\right]_{x=0}$$

$$= \frac{Bk\, m(T_s - T'_w)}{\sinh ml} \quad \text{from (10).}$$

$$= \frac{Bk\, m(T - T'_w)}{(1 + \epsilon)\sinh ml} \quad \text{from (12).}$$

This expression for $(dQ/dt)_{x=0}$ can now be substituted into (7):

$$(h_v L\bar{\beta}_w + h_c + h_r)_{sph}\, S(T'_w - T_w) = \frac{Bkm(T - T'_w)}{(1 + \epsilon)\sinh ml} . \qquad \ldots (13)$$

Put $T'_w - T_w = \Delta T_w$ and $h_c + h_r + h_v\bar{\beta}_w = h_e$

$$\frac{\Delta T_w}{T - T'_w} = \frac{Bkm}{S(1 + \epsilon)(h_e)_{sph}\sinh ml} .$$

This gives an approximate expression for the fractional decrease in wet-bulb depression if $T'_w \approx T_w$. If there is an appreciable difference between T'_w and T_w an iterative procedure is necessary but to a good approximation in all the cases met in practice

$$\frac{\Delta T_w}{T - T_w} = \frac{\Delta T_w}{T - T'_w} \cdot \frac{1}{1 + \dfrac{\Delta T_w}{T - T'_w}} . \qquad \ldots (14)$$

Expression (14) has been used in all the calculations as some of the values of ΔT_w obtained were appreciable.

The change in the psychrometer coefficient (see Part I or Wylie (1968b)) is given to a good approximation by the equation

$$dA = \frac{\Delta T_w}{T - T_w}(A + \bar{\beta}_w) \qquad \ldots (15)$$

where A is the psychrometer coefficient in the absence of stem heat conduction.

(a) Values of constants used in calculating the results

Thermal conductivity of glass	= 0.71 W m^{-1} K^{-1}
Thermal conductivity of mercury	= 7.95 W m^{-1} K^{-1}
Thermal conductivity of wick	= 0.63 W m^{-1} K^{-1}
Thermal conductivity of water	= 0.63 W m^{-1} K^{-1}
Radius of mercury thread in stem	= 0.75 mm
Radius of glass part of stem, lower part (generally covered with wick)	r = 3 mm
Radius of glass part of stem, upper part (generally free of wick)	r_d = 4 mm
Radius of stem including wick for conductive calculations,	r = 3.5 mm
Radius of stem including wick for convective heat transfer calculations,	r = 5.8 mm

(The wick is loose fitting and so presents a considerably larger effective radius for convective heat transfer than the radius of the neck and the thickness of the wick would suggest.)

Thermal conductivity of stem including wick	k = 0.75 W m^{-1} K^{-1}
Surface area of wet-bulb sphere,	S = 422 mm^2
Radiative heat transfer coefficient	h_r = 5.4 W m^{-2} K^{-1}

(b) Values of the heat transfer coefficients. The heat transfer coefficients for the wick-covered surfaces have been calculated using the psychrometer equation, e.g. (Wylie, 1968a)

$$h_e = (h_c + h_r)\,\frac{(A + \bar{\beta}_w)}{A}\,.$$

h_{cyl} has been deduced from the relations quoted by Wylie (1968c) for a cylinder in a state of forced convection by transverse airstream. The relation can be expressed approximately as

$$(Nu) = 0.56(Re)^{1/2}\,(Pr)^{1/3}$$

where (Nu) is the Nusselt number, (Re) the Reynolds and (Pr) the Prandtl numbers.

For a pressure of 1000 mb for normal temperatures this relation can be written

$$(h_c)_{cyl} = 3.31 \times 10^3 \left(\frac{V}{r}\right)^{1/2} \text{W m}^{-2}\ \text{K}^{-1}$$

where V is the airspeed expressed in metres per second and r is the radius of the cylinder expressed in millimetres. The appropriate value of r has been used for the wick-covered and non-wick-covered parts of the stem respectively; (Pr) has been put equal to 0.71, and $(h_c)_{sph}$ has been calculated from the results of Ranz and Marshall, given by Wylie (1968c) for a sphere

$$(Nu) = 2 + 0.60(Re)^{1/2}\,(Pr)^{1/3}\,.$$

From this equation $(h_c)_{sph}$ can be deduced for near-surface conditions as

$$(h_c)_{sph} = \frac{12}{R} + 2.51 \times 10^3 \left(\frac{V}{R}\right)^{1/2} \text{ W m}^{-2} \text{ K}^{-1} ;$$

R is expressed in millimetres and V in metres per second.

11.3. *Results*

Measurements at five outstations have shown that the effective length of the extension of the wick up the stem varies relatively little from wet bulb to wet bulb if the average length of the extension round the circumference is taken. This length was 2.7 mm. However, the effective length around the circumference of the neck of a single wet bulb may typically be estimated to vary from as little as 1 mm to as much as 4.5 mm, emphasizing the extremely irregular nature of the wet-bulb body.

Figure 13 shows the predicted psychrometer coefficient at $\overline{T + T_w} = 18°C$ using the corrected Kondo values as a reference for conditions of no heat transfer, to which have been added the estimates of dA. Figure 14 shows how the psychrometer coefficient may be expected to vary with $\overline{T + T_w}$ for selected mean ventilation rates while Figure 15 shows how the percentage decrease in the wet-bulb depression $\Delta T_w / (T - T_w)$ varies with $\overline{T + T_w}$. Figure 13 illustrates the good agreement between the theory and experiment in predicting the shape of the curve of the change in psychrometer coefficient with ventilation rate.

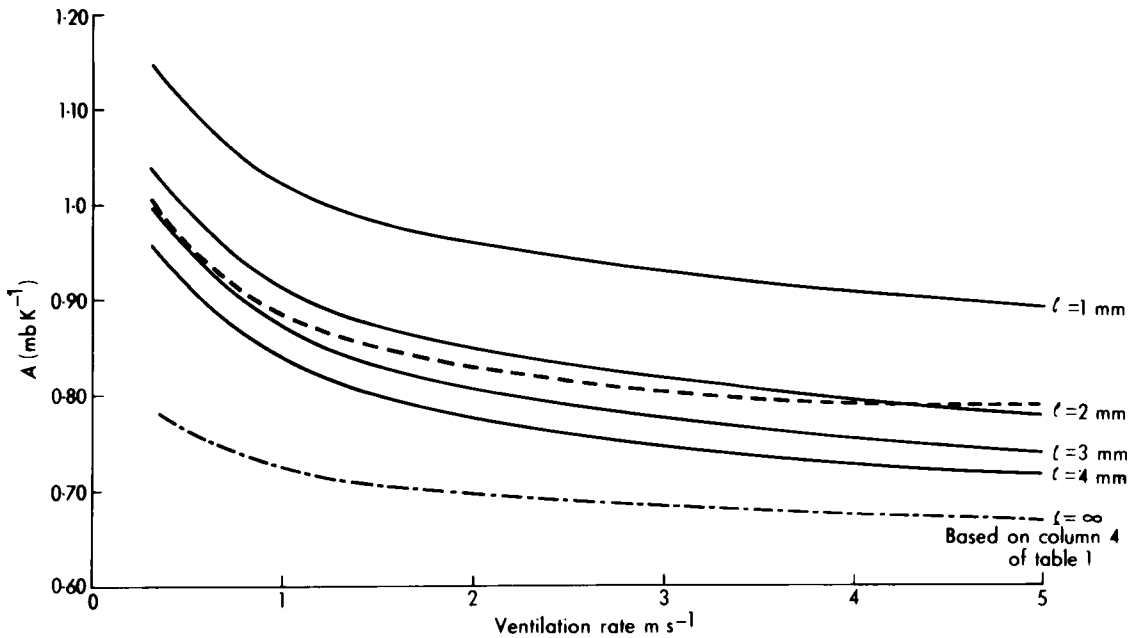

FIGURE 13. Predicted psychrometer coefficient – mercury-in-glass thermometer with muslin cap, $\overline{T + T_w} = 18°C$

Solid lines show predicted values, dashed lines show experimental value (l = 2.7 mm), and dots and dashes show values based on Column 4 of Table IV (no stem heat conduction).

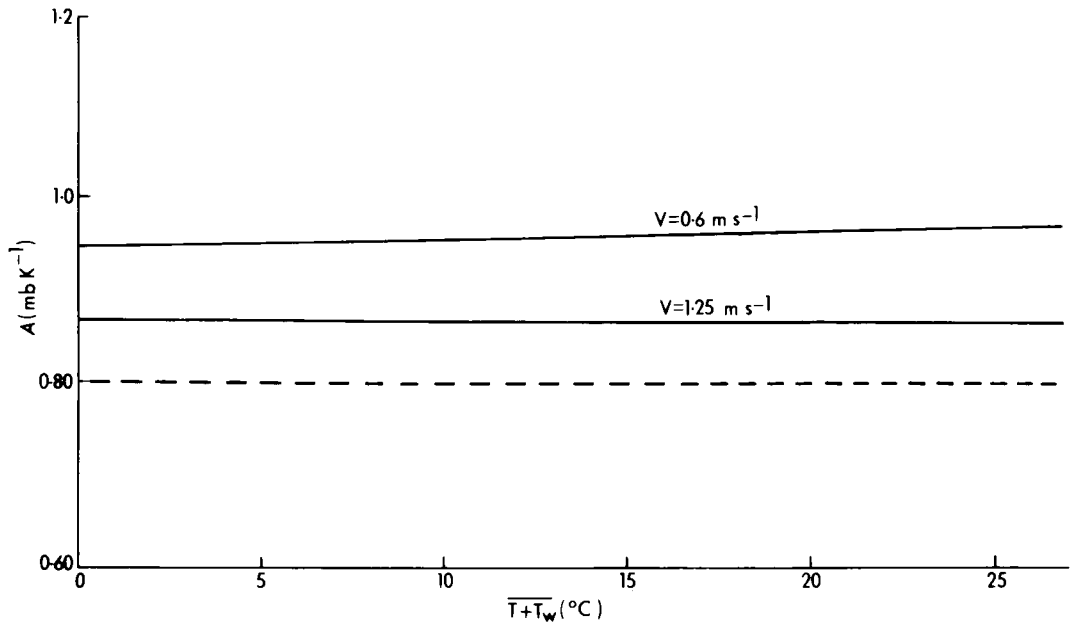

FIGURE 14. Predicted psychrometer coefficient as a function of the mean of the dry and wet bulb temperatures for the mercury-in-glass thermometer with muslin cap (l = 2.5 mm)

The dashed line denotes the coefficient in *Hygrometric Tables*.

The theoretical curves indicate that the experimental values of the psychrometer coefficient are consistent with a value of l = 2.5 mm, in close agreement with its estimated mean value at outstations. Figure 14 shows that, despite the stem heat conduction, the psychrometer coefficient remains virtually invariant with $\overline{T + T_w}$. Figure 15 predicts that the percentage decrease in wet-bulb depression due to stem heat conduction for l = 2.5 mm falls from over 16 per cent at $\overline{T + T_w}$ = 0°C to 7 per cent at $\overline{T + T_w}$ = 27°C. However, the *Hygrometric Tables* (Meteorological Office, 1964) use of A = 0.799 mb K^{-1} in fact accounts for some of this decrease (curves C and D) the amount depending on the assumed ventilation rates (see Part I for an explanation of the ventilation rates chosen for discussion). Curves E and F illustrate the predicted percentage decrease in wet-bulb depression which is not accounted for. At 0.6 m s^{-1} this varies from over 12 per cent at $\overline{T + T_w}$ = 0°C to under 6 per cent at $\overline{T + T_w}$ = 27°C, so that the unaccounted for percentage decrease in wet-bulb depression depends markedly on wet-bulb temperature, being least at high wet-bulb temperatures. These curves should be compared with Figure 3 in Part I. As described in Part I, an attempt was made to decrease the heat conduction effect by use of an aircraft psychrometer wick tied to the muslin cap. This was called the long-wick thermometer. It was not possible to eliminate stem heat conduction entirely as the upper part of the stem is enclosed by a sheath. The next section describes the attempt to predict the residual stem conduction of this thermometer and to compare it with the experimental results.

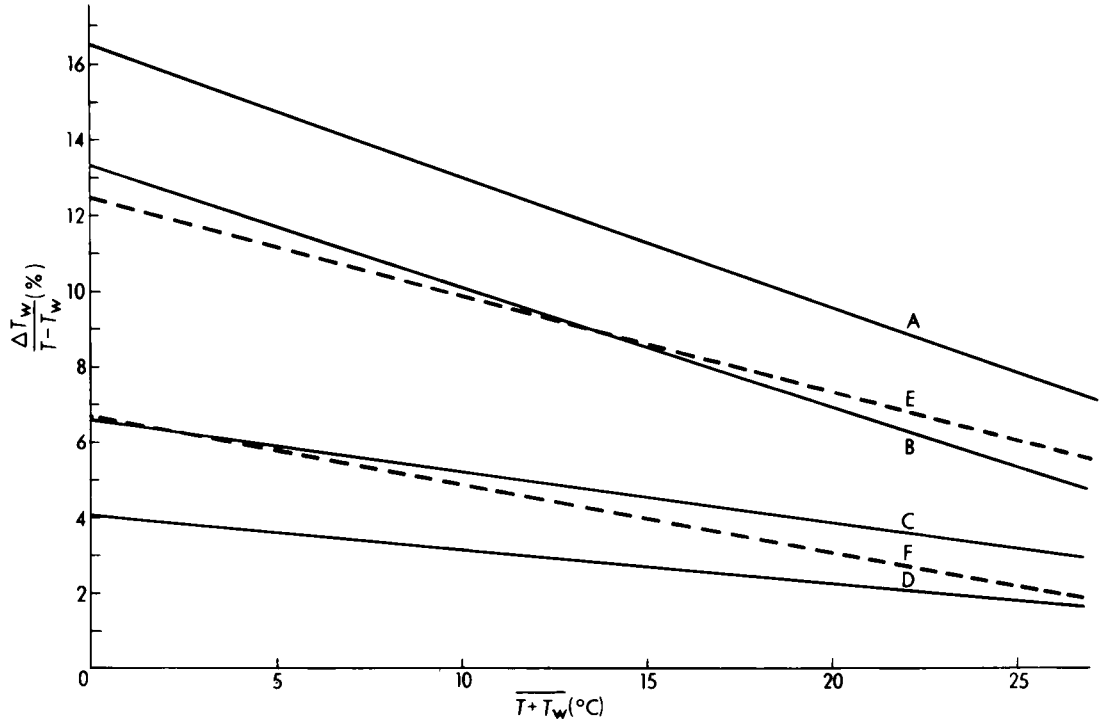

FIGURE 15. Variation with $\overline{T+T_w}$ of mercury-in-glass thermometer with muslin cap – predicted percentage decrease in wet-bulb depression

A Mean aspiration rate 0.6 m s^{-1}, l = 2.5 mm;
B Mean aspiration rate 1.25 m s^{-1}, l = 2.5 mm;
C Mean aspiration rate 1.25 m s^{-1}, A = 0.80 mb K^{-1};
D Mean aspiration rate 0.6 m s^{-1}, A = 0.80 mb K^{-1};
E Curve A minus curve D;
F Curve B minus curve C.

12. STEM HEAT CONDUCTION IN THE MERCURY-IN-GLASS THERMOMETER WITH LONG WICK

12.1. *General*

The same equation has been used as in preceding sections but with the following modifications to the physical constants and conditions.

(a) The wick closely adhered to the neck all the way along its length of 0.5 cm. This decreased the effective radius of the wick for convective heat transfer and slightly altered the values of h_e.

(b) The non-wick-covered stem is completely enclosed by the sheath, so it is assumed to be in a state of natural rather than forced convection. Thus values of $(h_c + h_r)_{cyl}$ appropriate to this state have been used. The equation for estimating the natural convection coefficient h_c has been taken from McAdams (1954), who quotes a simplified but accurate formula. Re-expressed in SI units this is

$$h_c = 9.3\left(\frac{\Delta T}{2r_d}\right)^{1/4} \text{ W m}^{-2} \text{ K}^{-1}$$

where ΔT is the mean temperature difference between the stem and the surrounding air and r_d is the radius of the stem expressed in millimetres. Strictly this equation is applicable to vertical pipes over 1 foot in length but as the equation for vertical plates less than 1 foot long is almost identical, the use of this equation is not likely to cause much error. Harder to predict is the value of ΔT which has been taken as typically 2°C but h_c varies little with ΔT. In Part I it was described how the outer sheath was covered with wick. This does not prevent stem heat conduction but does effectively eliminate conduction from the outer sheath which may well have been more important than the stem conduction as the sheath is relatively massive. Thus the outer sheath does not enter into the calculation.

(c) The effective radius of the sheath-covered stem used in the equations has been set to the radius where it joins the neck just inside the sheath.

(d) List of constants differing from those used in muslin-cap thermometer constants.

$$r_d = 4 \text{ mm}$$
$$l = 5 \text{ mm}$$
$$r = 3.4 \text{ mm}.$$

12.2. *Results*

The predicted increase in the psychrometer coefficient is a little less than experiment indicates. However, the experiment showed only a slight increase so the prediction of the

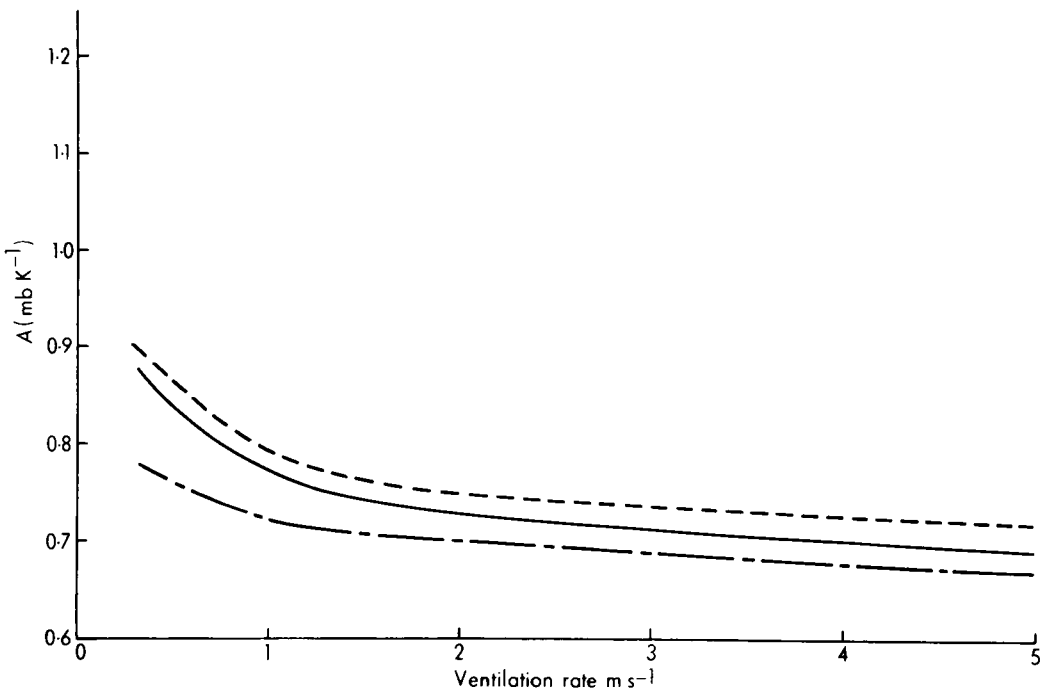

FIGURE 16. Predicted psychrometer coefficient for mercury-in-glass thermometer with long wick, $\overline{T + T_w} = 18°C$

Solid lines denote predicted values; dashed lines denote experimental values; dots and dashes indicate values based on Column 4 of Table IV (no stem heat conduction).

total psychrometer coefficient is still very close to that observed. The fact that the heat conduction is similar to that used in the *Hygrometric Tables* is satisfactorily accounted for. From Figure 6 the predicted coefficients are 0.82 mb K^{-1} at 0.6 m s^{-1} and 0.75 mb K^{-1} at 1.25 m s^{-1}. Values of $\Delta T_w/(T - T_w)$ are not shown since they are similar to those assumed using A = 0.799 mb K^{-1}. Thus (using the present tables) it is confirmed that there is no significant increase or decrease in the wet-bulb depression. The long-wick thermometer can therefore be used without significant error when the mean ventilation rate is between 0.6 m s^{-1} and 1.25 m s^{-1} in conjunction with the present tables.

13. STEM HEAT CONDUCTION IN THE PLATINUM RESISTANCE THERMOMETER

13.1. *General*

It was desired to confirm theoretically that, as normally used, there is no significant heat conduction down the wet-bulb stem. More importantly, it was required to predict how the platinum resistance thermometer could be made to have the same psychrometer coefficient, under given conditions of ventilation, as either of the types of mercury-in-glass thermometer described in previous sections. (See Part I for a description of the experiments.)

The constructional details of the thermometer are complex. It consists of a stainless steel sheath and contains an aluminium thimble in the bottom 5 cm of its length, inside which an alumina cylinder is placed. In the lower half of the alumina cylinder is placed the platinum wire helix, sensitive to temperature. The thermometer sometimes has a groove in the stainless steel about 45 mm above the bottom of the thermometer. This should have comparatively little effect on the sheath heat conduction as the stainless steel is underlain by a comparable thickness of aluminium of much higher thermal conductivity. This groove has been neglected in the following treatment and the wet bulb treated as a smooth cylinder. The thermometer is used with a thick tubular wick which extends into a reservoir beneath it. It is assumed below that there is no heat conduction up the extension of the wick beneath the thermometer and the water reservoir several tens of millimetres away. Figure 17 shows the model of the resistance thermometer system used in the following calculations.

The resistance thermometer is considered to be divided into four sections for the purposes of the model.

(a) *Thermometer bulb.* That part of the thermometer containing the temperature-sensitive element.

(b) *Lower stem* – length L. That part of the thermometer stem containing the alumina cylinder inside the aluminium thimble situated above the bulb and covered in wick.

(c) *Upper stem, lower section* – length l. Either (i) the remainder of the aluminium and alumina ceramic filled portion of the stem not covered in wick, or (ii) if all the aluminium and alumina ceramic filled part of the thermometer is wick-covered, that part of the stainless steel section which is wick covered.

(d) *Upper stem, upper section.* The remainder of the stainless steel section which is not covered by wick.

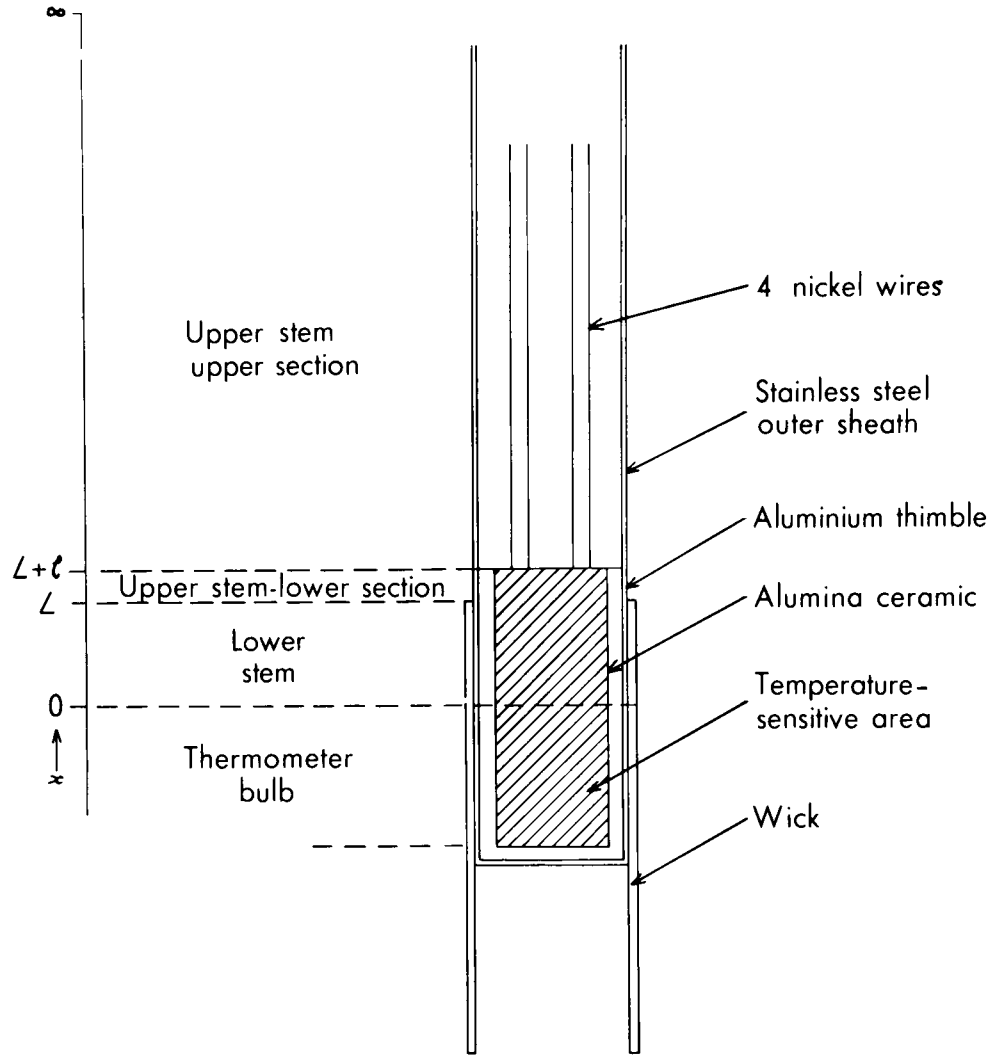

FIGURE 17. Model of platinum resistance thermometer

13.2. *Theoretical model*

An equation for the increase in psychrometer coefficient and decrease in wet-bulb depression due to stem heat conduction similar to that for the mercury-in-glass thermometer wet bulb is derived as follows.

Let θ be lower stem temperature,

θ' be upper stem, lower section temperature,

θ'' be upper stem, upper section temperature,

T'_w be the temperature at $x = 0$,

T''_w be the temperature at $x = L$,

T'''_w be the temperature at $x = L + l$, and

T be the dry-bulb temperature.

A similar analysis for the mercury-in-glass thermometer gives for the rate of change of temperature of the lower stem

$$\frac{d\theta}{dx} = m(T''_w - T'_w)\frac{\cosh mx}{\sinh mL} \quad , \qquad \ldots (16)$$

similarly for the rate of change of temperature of the upper stem, lower section

$$\frac{d\theta'}{dx} = m'(T'''_w - T''_w)\left\{\frac{\exp(m'x) + \exp(m'(2L - x))}{\exp(m'(L + l)) - \exp(m'(L - l))}\right\} \qquad \ldots (17)$$

and for the rate of change of temperature of the upper stem, upper section

$$\frac{d\theta}{dx} = m''(T - T'''_w)\left\{\frac{\exp(-m''x)}{\exp(-m''(L + l))}\right\} \qquad \ldots (18)$$

where $m = \left[\left(\frac{A + \bar{\beta}_w}{A}\right)(h_c + h_r)_{cyl}\left(\frac{2\pi r}{Bk}\right)\right]^{1/2}$ for lower stem,

$m' = \left[(h_c + h_r)_{cyl}\left(\frac{2\pi r_d}{B'_1 k'_1}\right)\right]^{1/2}$ for upper stem, lower section assuming condition (i) applies,

$m' = \left[\left(\frac{A + \bar{\beta}_w}{A}\right)(h_c + h_r)_{cyl}\left(\frac{2\pi r}{B'_2 k'_2}\right)\right]^{1/2}$ for upper stem, lower section assuming condition (ii) applies.

$m'' = \left[(h_c + h_r)_{cyl}\left(\frac{2\pi r_d}{B'' k''}\right)\right]^{1/2}$ for upper stem, upper section.

$h_c + h_r$ are evaluated for r_d or r as appropriate, a single prime represents the upper stem and a double prime the lower. Suffixes 1 and 2 represent the values of B and k appropriate to conditions (i) and (ii) respectively.

An equation can now be written for the heat transfer at the wet bulb in a similar way to equation (7) for the mercury-in-glass thermometer

$$(h_v)_{cyl} L\bar{\beta}_w S(T'_w - T_w) = (h_c + h_r)_{cyl} S(T_w - T'_w) + \left(\frac{dQ}{dt}\right)_{x=0} \qquad \ldots (19)$$

where S is the surface area of the cylindrical wet bulb enclosing the thermometer element.

Now $$\left(\frac{dQ}{dt}\right)_{x=0} = kB\left(\frac{d\theta}{dx}\right)_{x=0}$$

where k is the thermal conductivity of the lower stem and B the cross-sectional area.

From (16) is derived for the lower stem at $x = 0$

$$\left(\frac{d\theta}{dx}\right)_{x=0} = \frac{kBm(T''_w - T'_w)}{\sinh mL} \quad \ldots (20)$$

which, when substituted into (19) gives

$$(h_v L\bar{\beta}_w + h_c + h_r)_{cyl} S(T'_w - T_w) = \frac{kBm(T''_w - T'_w)}{\sinh mL} .$$

Rewriting $(h_v L\bar{\beta}_w + h_c + h_r)_{cyl}$ as h_e and putting $T'_w - T_w = \Delta T_w$,

$$\Delta T_w = \frac{kBm(T''_w - T'_w)}{(h_e)_{cyl} S \sinh mL} . \quad \ldots (21)$$

This equation is not immediately useful as T''_w is unknown. The equation must be represented in terms of $T - T'_w$.

Now at $x = L$ by continuity of heat conduction

$$kB\left(\frac{d\theta}{dx}\right)_{x=L} = k'B'\left(\frac{d\theta'}{dx}\right)_{x=L}$$

In the following argument k' and B' will be written whenever $k'_1B'_1$ or $k'_2B'_2$ are alternatives. Substituting for $(d\theta/dx)_{x=L}$ and $(d\theta'/dx)_{x=L}$,

$$\frac{kBm(T''_w - T'_w)}{\tanh mL} = \frac{k'B'm'(T'''_w - T''_w)}{\sinh m'l} .$$

On rearrangement this gives

$$T''_w - T'_w = \frac{T'''_w - T'_w}{1 + Q} \quad \ldots (22)$$

where

$$Q = \frac{kBm \sinh m'l}{k'B'm' \tanh mL} .$$

Now at $x = L + l$ by continuity of heat conduction

$$k'B'\left(\frac{d\theta'}{dx}\right)_{x=L+1} = k''B''\left(\frac{d\theta''}{dx}\right)_{x=L+1}$$

Substituting and rearranging gives the result that

$$\frac{T - T'_w}{1 + QR + Q} = T''_w - T'_w \quad \ldots (23)$$

where

$$R = \frac{k'B'm'}{k''B''m'' \tanh m'l} .$$

Equation (22) can then be rewritten by substituting equation (23) to obtain the final expression for the decrease in wet-bulb depression

$$\frac{\Delta T_w}{T - T'_w} = \frac{kBm}{(h_e)_{cyl} S(1 + QR + Q) \sinh mL} \quad \ldots (24)$$

This equation can be corrected in a similar way to the mercury-in-glass expression if T'_w is appreciably different from T_w.

Finally the expression for the increase in the psychrometer coefficient is as before

$$dA = (A + \beta_w) . \frac{\Delta T_w}{T - T_w}$$

(a) Numerical values of the parameters

$r = 3.7$ mm $\qquad r_d = 3.2$ mm

$S = 590$ mm^2

$k = 62.8$ W m^{-1} K^{-1} $\qquad k'_2 = 18.4$ W m^{-1} K^{-1}

$k'_1 = 83.7$ W m^{-1}K^{-1} $\qquad k'' = 10.9$ W m^{-1}K^{-1}

$B = 43$ mm^2 $\qquad B'_2 = 23$ mm^2

$B'_1 = 32$ mm^2 $\qquad B'' = 13$ mm^2

Length of aluminium thimble = 51 mm

Radius of nickel wires (4 wires) = 0.25 mm

Cross-sectional area = 0.8 mm^2

Cross-sectional area of stainless steel sheath = 12 mm^2

Cross-sectional area of aluminium thimble = 9.5 mm^2

Cross-sectional section of alumina ceramic = 9.5 mm^2

Thermal conductivity of stainless steel (18/8 type) = 15.5 W m^{-1}K^{-1}

Thermal conductivity of nickel = 87.9 W m^{-1}K^{-1}

Thermal conductivity of aluminium = 238.6 W m^{-1}K^{-1}

Thermal conductivity of alumina ceramic = 20.9 W m^{-1}K^{-1}

Other parameters have the values indicated in section 11.2(a). The thermal conductivity of the thermometer sheath and its cross section above the level of the thermometer bulb have been evaluated including the nickel wires as if they were part of the sheath by adding their cross section, with appropriate thermal conductivity, to give a parallel path to that of the sheath. For this a composite cross section and thermal conductivity was calculated. The nickel wires contribute relatively little to the overall conduction so this approximation does not introduce significant errors.

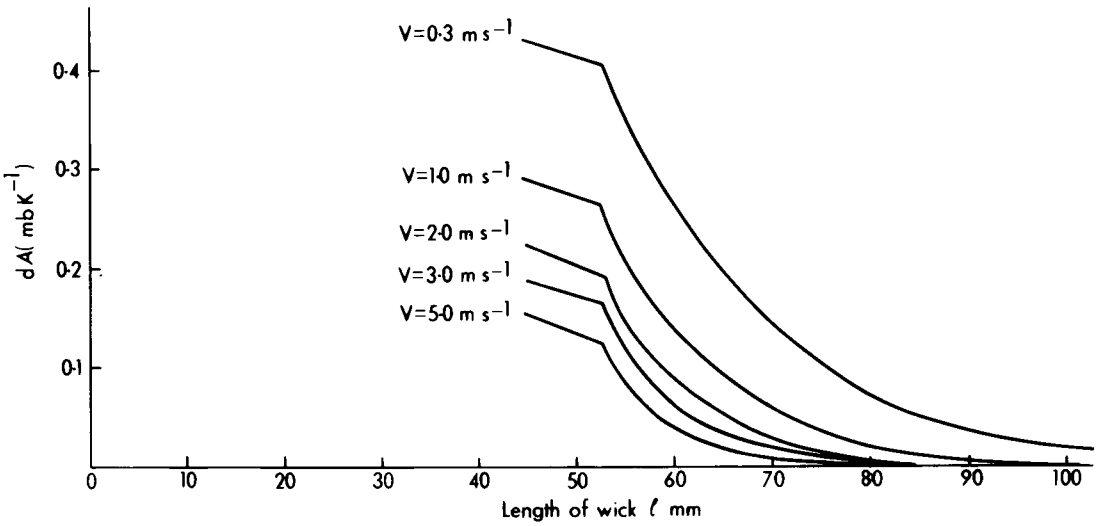

FIGURE 18. Calculated increase in psychrometer coefficient at $\overline{T + T_w}$ = 18°C, for resistance thermometer Mk 2, due to stem heat conduction

13.3. *Results*

Figure 18 shows the computed values of dA for $\overline{T + T_w}$ = 18°C for various ventilation rates and wick lengths l'. The wick length is defined as the distance between the base of the thermometer and the top of the tubular wick on the stem. The diagram confirms that stem heat conduction is negligible for a 100-mm wick. Thus the experimental values quoted in Part I have been used to estimate the psychrometer coefficient of the resistance thermometer in the absence of stem heat conduction and these estimated values are listed in Table V.

The values of dA for other wet-bulb temperatures are similar to those at $\overline{T + T_w}$ = 18°C and are not shown here. Figure 19 shows the psychrometer coefficient of the resistance thermometer for various wick lengths at $\overline{T + T_w}$ = 18°C compared to the experimental values. The calculated values are seen to be a little lower than the experimental ones at high ventilation rates but in better agreement at lower rates. At V = 0.6 m s^{-1} the wick, according

TABLE V. *Measured psychrometer coefficient of resistance thermometer with 100-mm wick*

Ventilation rate	A
$m\ s^{-1}$	$mb\ K^{-1}$
0	1.05
0.3	0.80
1.0	0.72
2.0	0.68
3.0	0.67
5.0	0.67

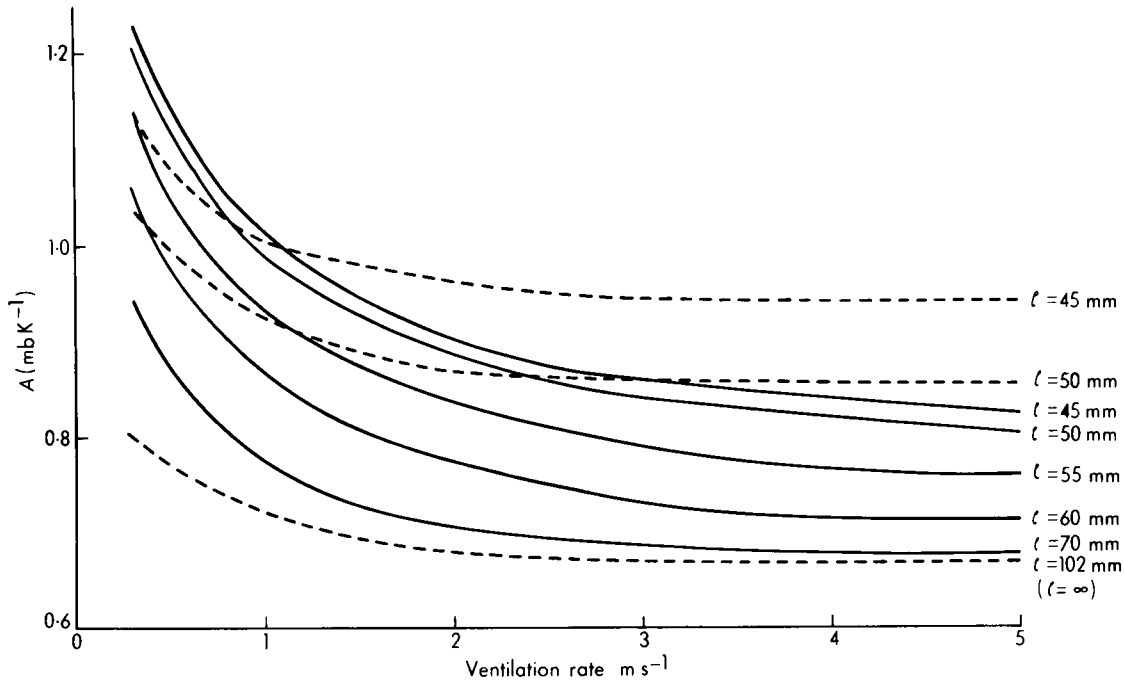

FIGURE 19. Psychrometer coefficient at $\overline{T + T_w}$ = 18°C of resistance thermometer Mk 2 for various wick lengths

Solid lines indicate calculated values; dashed lines show experimental values.

to the calculations, would have to be about 80 mm long to produce a psychrometer coefficient equal to the value used in the *Hygrometric Tables* while at V = 1.25 m s^{-1} the wick length would be about 64 mm long.

It is also of interest to consider how the resistance thermometer wick would need to be modified to give it a psychrometer coefficient as nearly as possible equal to the mercury thermometer with muslin cap and the mercury thermometer with the long wick. Figures 20 and 21 show the results. In each diagram the experimental curve for the mercury thermometer is plotted together with the calculated curve for the resistance thermometer for a particular wick length, at V = 0.8 m s^{-1} approximately. The shape of the two curves is slightly different in both Figures 20 and 21 so it is not possible to make the coefficients equal at all ventilation rates. This of course may reflect the inadequacies of the theory; but the apparent difference in the psychrometer coefficient at other ventilation rates is not serious, especially as no account up to now has been taken of changes in ventilation rates in a naturally aspirated psychrometer. Figure 20 shows that a wick length of 58 mm on the resistance thermometer would make it have a similar psychrometer coefficient to the mercury thermometer with muslin cap. Figure 21 shows that a wick length of 68 mm would result in a resistance wet bulb similar in performance to that of the mercury thermometer with long wick.

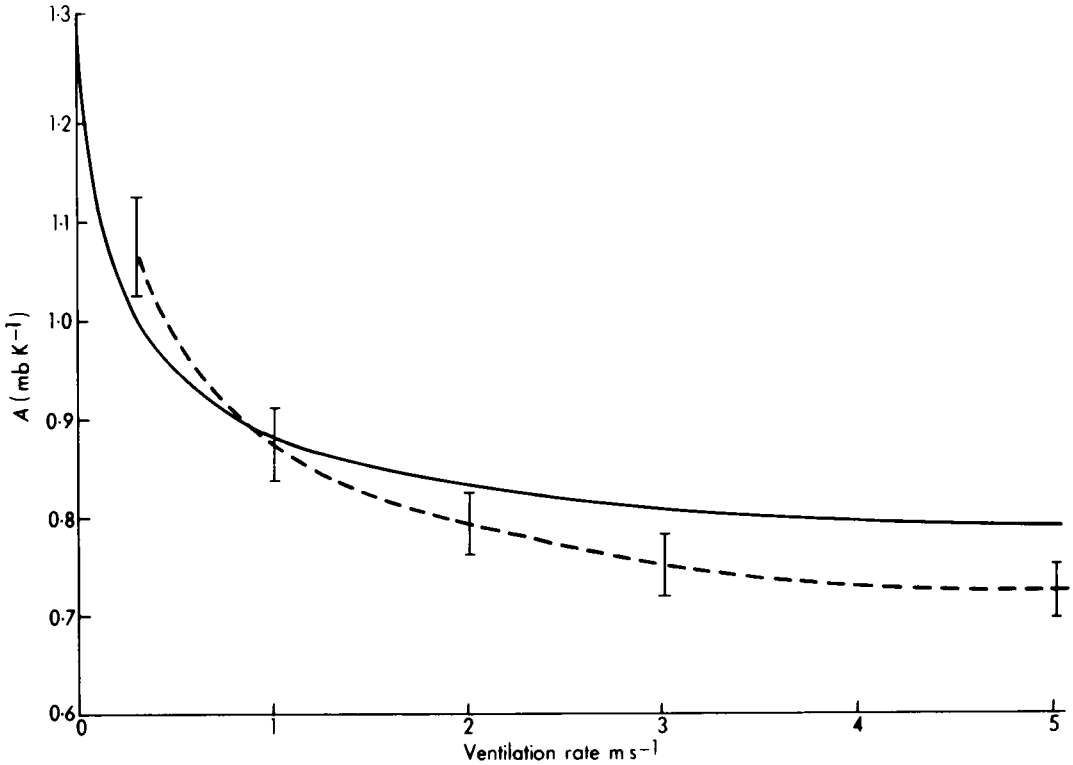

FIGURE 20. Comparison of the psychrometer coefficients of the mercury muslin-cap thermometer (measured) and the resistance thermometer with a wick length of 58 mm (calculated), $\overline{T + T_w} = 18°C$

Solid lines are muslin-cap thermometer; dashes are resistance thermometer; vertical bars denote the effect of tolerance of ± 3 mm on resistance thermometer wick length. Top of bar represents shorter wick length.

14. CONCLUSIONS

Calculations have confirmed that the measured excess of the psychrometer coefficients of the mercury thermometer in the muslin cap over theoretical estimates are a result of appreciable stem heat conduction. In Part I it was shown that the measured coefficients of the mercury thermometer with muslin cap were also higher than was consistent with use of the published coefficients in the *Hygrometric Tables.* The calculations confirm, however, that the psychrometer coefficient of the resistance thermometer with a wick length of 100 mm is for practical purposes unaffected by stem heat conduction so that the resistance thermometer behaves as an ideal wet bulb. The difference between the two thermometers can be largely removed and both thermometers made as consistent as is possible with the *Hygrometric Tables* (for a naturally aspirated psychrometer using a fixed psychrometer coefficient) by using a 68-mm wick on the resistance thermometer and by the mercury thermometer with a long wick. It is important to realize, however, that the errors resulting from poor ventilation, particularly in near-calm conditions can be considerably larger than any of the differences described in this paper. However, the above procedure removes the existing systematic errors in the best way, given that the psychrometer is of an unaspirated type.

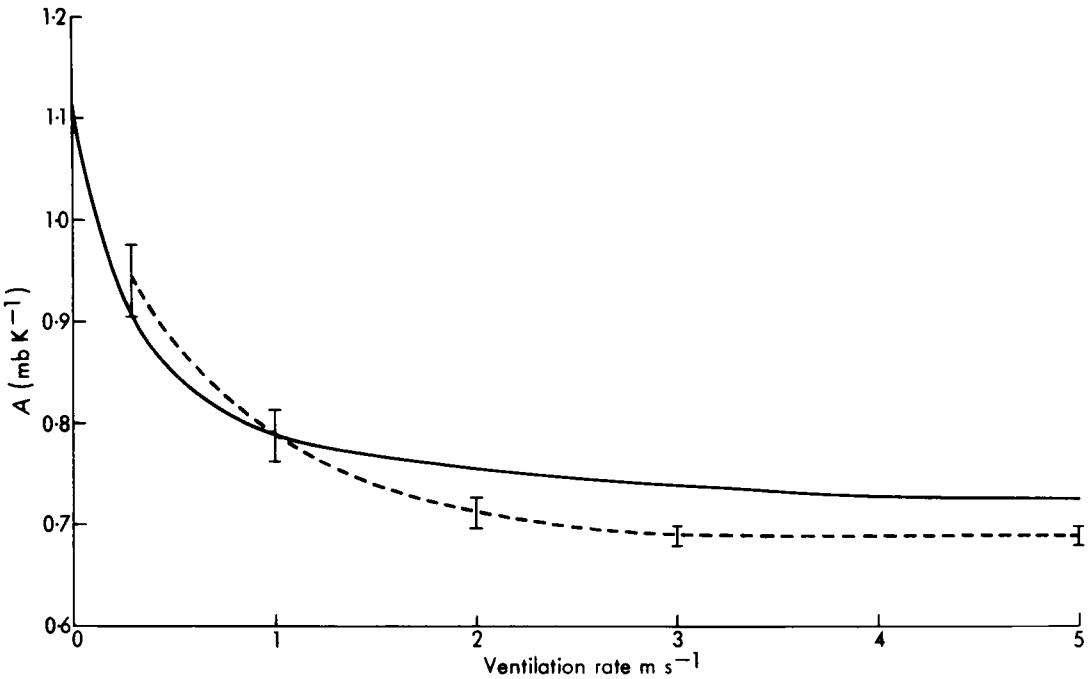

FIGURE 21. Comparison of psychrometric coefficient (measured) of long-wick thermometer with resistance thermometer having 58 mm wick (calculated), $\overline{T+T_w}=18°C$

Solid lines are long-wick thermometer, dashed lines are resistance thermometer; vertical bars show the effect of tolerance of ± 3 mm on resistance wick length.

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BIBLIOGRAPHY

BULTOT, F. and DUPRIEZ, C. L.; 1971, Comparaison d'instruments de mesure de l'humidité de l'air sous abri. *Arch Met Geophys Bioklim, Wien,* **19B**, pp. 53–67 (translation available in the Meteorological Office Library, Bracknell).

CLARKSON, L. S.; 1971, A comparison of the performance of mercury-in-glass and electrical resistance thermometers in the screen at Brize Norton (unpublished, copy available in the Meteorological Office Library, Bracknell).

FOLLAND, C. K. and SPARKS, W. R.; 1976, A two-pressure humidity generator for calibrating electrical hygrometers used in meteorology. *J Phys,* E, *Scient Instrum,* **9**, pp. 112–116.

KONDO, J.; 1967, Psychrometric constant for different sizes of the wet thermometer. *Scient Rep Tohuka Univ, Geophys,* **18**, pp. 125–137.

MACDOWALL, J.; 1956, An electrical dewpoint hygrometer suitable for routine meteorological use (unpublished, copy available in the Meteorological Office Library, Bracknell).

MARVIN, C. F.; 1941, Psychrometric tables for obtaining the upper pressure, relative humidity and temperature of the dew point. Washington, Weather Bureau, W.B. No. 235.

MCADAMS, W. H.; 1954, Heat transmission. McGraw-Hill Book Company, 3rd edition.

METEOROLOGICAL OFFICE; 1964a, Hygrometric Tables, Part III, Aspirated psychrometer readings – Degrees Celsius. 2nd edition. HMSO.

METEOROLOGICAL OFFICE; 1964b, Hygrometric Tables, Part II, Stevenson screen readings. 2nd edition. HMSO.

PAINTER, H. E.; 1970, A recording resistance psychrometer. *Met Mag,* **99**, pp. 68–75.

PAINTER, H. E.; 1973, The performance of wet-bulb thermometers in the large thermometer screen. *Met Mag,* **102**, pp. 212–215.

PERNTER, J. M.; 1903, Jelineks Psychrometer-Tafeln, pp. V–XIII. Leipzig.

REGNAULT, M. V.; 1845, Etudes sur l'hygrométrie, pp. 96–112. Paris (translation available in the Meteorological Office Library, Bracknell).

SHEPPARD, P. A., TRIBBLE, D. T. and GARRETT, J. R.; 1972, Studies of turbulence in the surface layer over water (Lough Neagh). Part I. Instruments, programme, profiles. *Q J R Met Soc,* **98**, pp. 627–641.

WYLIE, R. G.; 1968a, Résumé of knowledge of the properties of the psychrometer. Melbourne, CSIRO. Report No. PIR–64.

WYLIE, R. G.; 1968b, Tables of some quantities derived from the psychrometer equation. Melbourne, CSIRO. Report No. PIR-61.

WYLIE, R. G.; 1968c, Heat and vapour transfer data for calculations relating to psychrometers. Melbourne, CSIRO. Report No. PIR-62.

YOSHITAKE, M. and SHIMIZU, I.; 1965, Experimental results of the psychrometer constant. E. RUSKIN (editor), Humidity and moisture, Vol. 1. New York, Reinhold Publishing Corporation, pp. 70–75.

ZOBEL, R. F.; 1965, An example of ventilation error in the dry-bulb and wet-bulb psychrometer. *Met Mag,* **94**, pp. 161–166.

APPENDIX

1. *Relationship between the error in A and the error in $T - T_w$*

$$e - e_w = -A(T - T_w) \qquad \ldots (A.1)$$

is the well-known expression for the psychrometer 'constant', where T is dry-bulb temperature, T_w is wet-bulb temperature, A is the psychrometer coefficient, e is the vapour pressure of the air and e_w is saturated vapour pressure at wet-bulb temperature. Here we assume the pressure term is incorporated into A. Differentiating $(T - T_w)$ with respect to A,

$$\frac{\partial}{\partial A}(T - T_w) = \frac{(e - e_w)}{A^2} + \frac{1}{A}\frac{\partial e_w}{\partial A}; \qquad \ldots (A.2)$$

now

$$\frac{\partial e_w}{\partial A} = \frac{\partial e_w}{\partial T_w}\frac{\partial T_w}{\partial A} .$$

If we also define

$$\frac{\partial e_s}{\partial T_w} = \bar{\beta}_w \approx \frac{\partial e_w}{\partial T_w} \qquad \ldots (A.3)$$

then

$$-\frac{\partial}{\partial A}(T - T_w) = \frac{-e - e_w}{A^2} - \frac{\bar{\beta}_w}{A}\frac{\partial T_w}{\partial A} .$$

For a fixed value of T this becomes

$$-\frac{\partial}{\partial A}(T - T_w) = \frac{-(e - e_w)}{A^2} + \frac{\bar{\beta}_w}{A}\frac{\partial}{\partial A}(T - T_w) .$$

Rearranging, it can be shown that

$$\frac{\partial(T - T_w)}{T - T_w} \Big/ \frac{\partial A}{A} = \frac{-A}{A + \bar{\beta}_w} \qquad \ldots (A.4)$$

This equation gives the proportional change in $T - T_w$ compared with the proportional change in A. As an example:

At T_w is 0°C, $\bar{\beta}_w$ is 0.45 and if A is near 0.8

$$\frac{\partial(T - T_w)}{T - T_w} \Big/ \frac{\partial A}{A} = -0.65$$

At T_w is 25°C, $\bar{\beta}_w$ is 1.88 and if A is near 0.8

$$\frac{\partial(T - T_w)}{T - T_w} \Big/ \frac{\partial A}{A} = -0.30 .$$

This means for example a 10 per cent increase in A corresponds to a 6.5 per cent decrease in $T - T_w$ at $T_w = 0°C$ and a 3 per cent decrease in $T - T_w$ at $T_w = 25°C$.

2. *Relationship between the wet-bulb depression and the psychrometer coefficient*

Now $$\bar{\beta}_w = \frac{\partial e_s}{\partial T_w} \quad \text{from (A.3).}$$

Putting this in finite-difference form gives (where $e_s = e_w$)

$$\Delta e_w = \bar{\beta}_w \Delta T_w \cdot$$

Consider conditions at 100 per cent humidity; then $e_w = e_D$ and $T_w = T$ where e_D is saturation vapour pressure at the dry-bulb temperature T. Consider a reduction in relative humidity and a change in wet-bulb temperature ΔT_w

$$\Delta e_w = e_D - e_w \cdot$$

Now $$\Delta T_w = T - T_w$$

Thus $$e_D - e_w = \bar{\beta}_w (T - T_w) \cdot$$

Putting $e = Ue_D$ where U is relative humidity expressed as a fraction, and placing into equation (A.1), it can easily be shown that

$$T - T_w = \frac{(1 - U) e_D}{A + \bar{\beta}_w} \cdot \qquad \ldots (A.5)$$

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